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A CURRICULUM IMPLEMENTATION OF CREATIVE PROBLEM SOLVING IN JUNIOR HIGH SCHOOL MATHEMATICS

by

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A THESIS

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ABSTRACT

The purpose of the present study was to investigate the feasibility of implementing creative problem solving in junior high school mathematics curriculum. Four essential steps comprised the investigation:

(1) Adapting from Boychuk's creative problem solving model, a three process model of mathematical creativity based on the processes of sensitivity, redefinition and conjecturing was first conceptualized.

(2) This conceptual model was operationalized in terms of three characteristics of student responses to creative problem-situations which optimally reflect sensitivity, redefinition and conjecturing. The characteristics (fluency, diversity and rarity of ideas) served as scoring criteria for the creative tests employed.

(3) The Inventive Method (IM) was developed to "control" student learning activity by facilitating and rewarding inventive, divergent thinking. IM comprised three instructional phases: (a) Development of Concepts (DC) i.e. mastery of prerequisite mathematical content knowledge, via direct expository teaching; (b) Inventive Exercises (IE) to provide adequate creative problem solving experience; (c) Inventive Discussion (ID) to stimulate and reward creative thinking. For purposes of comparison, the Traditional Method (TM) was developed, comprising (a) the same DC phase, (b) Traditional Exercises (TE) i.e. traditional, convergent-

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type problems, and (c) Traditional Discussion (TD). Relevant instructional materials in the selected content domain (junior high motion geometry) were constructed for both IM and TM.

(4) Two Grade Eight classes in an Edmonton Separate school participated in the study. One regular mathematics teacher taught the experimental (N=30) and control (N=28) groups respectively through IM and TM for 6 weeks. Five pretests on students' mathematical achievement and mental ability were given. The degree of implementation of IM and TM was assessed using the Observer Rating Scale of Teacher Behavior. Students' terminal performance in traditional achievement, creative achievement and transferability were measured on four post-tests.

Complete data were obtained for 41 students (experimental N=20; control N=21). Multivariate analyses of 5 pretest measures showed that the two groups were comparable with respect to students' mathematical competency, Euclidean level of geometric maturity and mathematical creativity. Classroom observations and Observer Rating Scale data confirmed satisfactory implementation of IM and TM. Multivariate analyses of the post-test measures showed that (1) the IM group attained the same level of traditional, convergent achievement in motion geometry as the TM group, and (2) the IM group outperformed the TM group on two creative geometry tests and one transfer achievement test.

The major conclusion of the study is that through appropriate operationalization of Boychuk's model of creative problem solving, an instructional method for enhancing student

type problems, and (c) Traditional Discussion (TD). Relevant instructional materials in the selected content domain (junior high motion geometry) were constructed for both IM and TM.

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The major conclusion of the study is that through appropriate operationalization of Rychuk's model of creative problem solving, an instructional method for enhancing student

mathematical creativity could be developed and successfully implemented in a significant content domain of junior high school mathematics curriculum. The results also demonstrate that creative teaching can accomplish what conventional teaching is supposed to achieve, i.e. the learning of specific mathematical content by students. Furthermore, the creative method can better facilitate the utilization of knowledge learned in this specific content area to new mathematical situations.

If creative thinking is a desirable outcome of school mathematics curriculum, the study has shown that this end can be effectively attained using a certain strategy. The sequential approach adopted in developing the various phases of the Inventive Method of creative teaching provides, it is suggested, a valid and meaningful basis for future attempts to formulate a theory of creative instruction.

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CHAPTER I

INTRODUCTION TO THE PROBLEM

BACKGROUND

The surge of interest in the experimental study of creativity in educational psychology can justifiably be regarded as an advance in pedagogical theorizing. Reflective of such interest is the founding of the Creative Education Foundation in 1954, the Journal of Creative Behavior in 1967, and the many new programs designed to enhance creative problem solving skills in various school subjects (Roweton, 1973).

Advocates of creative thinking essentially argue that traditional forms of education tend to train students to produce the "right answer", with relative neglect of the kind of intellectual activity which explores new possibilities and search for original solutions to intellectual problems. Moreover, conventional examination performance too often depends on the regurgitation of accepted facts and opinions, or the mechanical application of learned rules and principles, hence stifling the creative potentiality of the students. Dissatisfaction with such a traditional view of the objectives of education and a narrow definition of intellectual ability has stimulated research into creative thinking.

The classical works of Guilford (1950, 1956, 1959, 1961, 1962), Getzels and Jackson (1962), and Torrance (1962, 1963, 1966)

among others, have provided substantial impetus to the view that creativity involves mental capacity apart from those measured by most of the existing standardized tests of mental ability. An additional source of motivation for the increasing interest in creativity is the growing awareness of the vast differences between traditional academic achievement and creative performance (McGannon, 1972: 7).

Although creativity has been hypothesized to exist in many different forms by different researchers, the various definitions of creative thinking do have a common core of agreement. It is generally assumed implicitly or explicitly that the capacity and propensity for creative behavior is a universal human trait (Maslow, 1959; Stein, 1974), and, similar to other mental abilities, is normally distributed throughout the population (Roweton, 1973: 1). Another assumption that underlies most research in creativity is that it is worthwhile to explore and understand creative thinking so that techniques can be developed to enhance the creativity of students.

In mathematics education, as in other subject areas, new developments have emphasized "inventiveness: reasoning creatively in mathematics" (Husen, 1967: 81) as one of the main objectives of school mathematics. The study of creative behavior in mathematical situations effectively began from the introduction of the "new mathematics" program into many school curricula and the application of the "discovery method" to classroom mathematics instruction. In the United States, the Madison Project (Davis, 1964), for example, has emphasized the creative informal exploration of problems by children. The Nuffield Foundation (1964) in the United Kingdom has

sponsored a Primary School Mathematics Project with the aim of fostering in children a critical and logical but also creative turn of mind through learning by discovery. Representative of the advocacy for creative mathematics instruction are the claims that:

> The discovery approach, in which the student is asked to explore a situation in his own way, is invaluable in developing creative and independent thinking in the individual (Cambridge Conference on School Mathematics, 1963: 17).

> Our knowledge about any subject consists of information and know-how. Know-how is ability to use information; of course, there is no know-how without some independent thinking, originality and creativity. Know-how in mathematics is the ability to do problems, to find proofs, to criticize arguments, to use mathematical language with some fluency, to recognize mathematical concepts in concrete situations.
> Everybody agrees that, in mathematics, know-how is more important, or even much more important, than mere possession of information. Everybody demands that the high school should impart to the students not only information in mathematics but know-how, independence, originality, creativity (Polya, 1965: 113).

In his attempt to summarize the psychological principles underlying the "new mathematics" movement in ten statements, Scott (1966: 15-16) included the following pair of principles:

> 5. The inductive approach or the discovery method is logically productive and should enhance learning and retention.
> 6. The major objective of a program is the development of independent and creative thinking processes.

A further important argument which comes from Bruner (1966: 158) is that "creativity" in education means more than "discovery" of known facts or knowledge. "Children, like adults, need reassurance that it is all right to entertain and express highly subjective ideas, to treat a task as a problem where you

<u>invent</u> an answer rather than <u>finding</u> one out there in the book or on the blackboard."

In short, underlying the research of such creativity theorists as Lankford (1959), Brune (1961), Hohn (1961), Kemeny (1961), Evans (1964), Bandet (1968), Willoughby (1968), Wheeler (1970a), Taylor-Pearce (1971), Boychuk (1974), Balka (1974) and Jensen (1976), is the premise and conviction that one of the important goals of mathematics education is to develop in students their ability to create new ideas in mathematical problem situations. As Wheeler (1970b: 150) sums up neatly this new direction in mathematics education:

> To see mathematics as invention rather than discovery, and as applicable to virtually any raw material whatever, can give us a different sense of its place in education.

STATEMENT OF THE PROBLEM

Concomitant with the growth of interest in the notion of creativity, recent years have seen an increasing number of studies dealing with creative thinking in school mathematics. However, most of these have been concerned either with the construction of instruments for measuring postulated mathematical creativity (Kilpatrick, 1969) or with the correlations between mathematical creativity and other relevant variables, such as mathematics achievement, intelligence, logical reasoning, personality traits and various psychological factors (Aiken, 1973). It has also been noted by Aiken (1973: 27-28) that studies with "special

instructional techniques designed to enhance either general creativity or mathematical creativity sometimes appear to have little, if any, effect on performance", and "significant differences in general creativity or mathematical creativity were not found." He concludes his methodological criticisms with these words:

> Various explanations for the negative results obtained in these studies and the low generalizability of the findings are possible: high initial levels of criterion performance, small sample sizes, failure to randomly assign subjects to subgroups, lack of proper control over extraneous variables, inadequate measuring instruments. With respect to the last item, the reliabilities and validities of psychological tests and inventories are typically lower in the early grades than further up the educational ladder (Aiken, 1973: 28).
>
> Many educational studies suffer from small and biased samples, inadequate controls, failure to meet the assumptions underlying specific statistical methods, and unreliable or invalid measuring instruments. In addition, the meaning of the concept of creativity is unclear and varies somewhat with the investigation and the technique (Aiken, 1973: 31-32).

Indeed, many educators and researchers are still debating the question whether creative thinking in school mathematics can be effectively taught. As McGannon (1972: 12) summarized on behalf of some pessimistic studies:

> Apparently, no one as yet knows how to design school experiences that will foster creative acquisition of information. Furthermore, it is still not determined which kinds of information can be learned more effectively by authority and which by creative means; nor has it been determined which students will do better by which means.

Not surprisingly, a similar state of affairs occurs in the area of "discovery learning". In his comprehensive analysis of the literature, Wittrock (1966) found that "discovery learning"

generally suffers from the same conceptual and methodological weaknesses. This has led in turn to inconsistent findings. According to Bishop and Levy (1968: 61), "the major problem is that of defining adequately the subject matter domain and the teaching methods", and "a great deal of the difficulty revolves around the language used to discuss and describe teaching methods -- it is at best vague, and at worst contradictory."

These shortcomings in research in instructional methods are unfortunately not limited to discovery learning. Many of the issues in "creative thinking" are as yet unstated in manageable form, and many of the findings of one study are often overthrown by another (Getzels and Dillon, 1973: 703). Entwistle and Nisbet (1972: 168) also note that the narrow scope of many of the constructed tests of creativity compared with the multiplicity of ways in which creative abilities may be utilized, makes the link between tests and performance extremely tenuous.

The purpose of the present study is fourfold:

(a) To select a well-established theoretical model of creative problem solving behavior in school mathematics, and adapt it to the present study through appropriate analyses and modifications;

(b) to operationalize this adapted theoretical model in terms of concepts based on related instructional theories, and thus establish practical guidelines for designing the instructional method and constructing instructional materials, which would enhance mathematical creativity in junior high students;

(c) to apply these guiding principles to a specific

subject content domain within the junior high school mathematics curriculum, and to design an inventive method of instruction with suitable instructional materials that would enhance students' ability to solve creative mathematical problems;

(d) to design appropriate evaluation instruments and procedures that would assess the instructional method as a process, as well as the outcomes of the instruction in terms of traditional achievement, creative achievement and transferability.

Specifically, the study seeks answers to the following questions:

(1) How can a theoretical model of creative problem solving in school mathematics be meaningfully adapted and translated into practical instructional guidelines which can guide classroom instruction in a specific subject matter domain?

(2) How can specific instructional procedures and materials be constructed that would train junior high students to solve mathematical problems in an inventive or creative manner?

(3) Do such creative pedagogical strategies and instructional materials, which encourage and reward inventive and divergent thinking of students in a specific subject content area, affect students' ability in solving both types of problems of that content domain, viz. traditional convergent type and inventive divergent type?

(4) Do such creative pedagogical strategies and inventive instructional materials affect students' ability to transfer learned mathematical knowledge in that specific content area to

novel problem-situations in other related but quite different areas of mathematics?

DEFINITIONS

The pertinent definitions of terms are stated briefly below. Further elaboration of these terms will be given in subsequent chapters.

Creativity. Creativity is defined conceptually in terms of the three processes of sensitivity, redefinition and conjecturing; and operationally in terms of fluency, diversity and rarity of ideas.

Divergent Thinking. Divergent thinking is a way of reasoning which emphasizes the generation of many appropriate ideas or solutions to a given problem-situation.

Convergent Thinking. Convergent thinking is a way of reasoning towards acquiring a single solution or closure to a problem-situation for which there is a known or a generally accepted answer.

Sensitivity. Sensitivity is a divergent thinking process which includes the ability to perceive deficiencies and shortcomings in a given problem-situation, and the ability to "see" possibilities or alternatives that lead to a variety of solutions.

Conjecturing. Conjecturing is a divergent thinking process which generates responses to problem-situations in the form of guesses or hypotheses.

Redefinition. Redefinition is a convergent thinking process

which includes the transformation of a problem-situation by reassociating previously unassociated elements of information to result in new combinations, and discarding previously adequate yet ineffective approaches to facilitate the perception and solutions of the given problem-situation.

Fluency. Fluency of ideas refers to the production of a large quantity of distinct appropriate solutions to a given problem-situation. The fluency score is a measure of the total number of these appropriate and distinct solutions.

Diversity. Diversity of ideas refers to the production of a large quantity of distinct categories of concepts reflected in the solutions produced. The diversity score is a measure of the total number of these "diversity categories".

Rarity. Rarity of ideas refers to the production of distinct and original categories of concepts reflected in the solutions produced. The rarity score is a measure of the infrequency or uncommon-ness of these "diversity categories". The more frequently a category is given within a sample of students, the lower the rarity score.

Inventive Method (IM). The Inventive Method of instruction consists of the following sequence of instructions: Development of Concepts, Inventive Exercises, and Inventive Discussion.

Traditional Method (TM). The Traditional Method of instruction consists of the following sequence of instructions: Development of Concepts, Traditional Exercises, and Traditional Discussion.

Development of Concepts (DC). This phase of instruction refers to the introduction of new mathematical concepts in a conventional direct expository approach according to the materials and instructions given in the Junior High Mathematics Program: Geometry -- Level Eight (Edmonton Separate School Board, 1975), hereinafter referred to as the Program.

Inventive Discussion (ID). This phase of instruction refers to the classroom discussion of the Inventive Exercises (IE) specially designed to contain both divergent and convergent types of problems.

Traditional Discussion (TD). This phase of instruction refers to the classroom discussion of the Traditional Exercises (TE) given in the Program. Such exercises emphasize convergent thinking.

ASSUMPTIONS

The present study is premised on four fundamental assumptions:

(1) that all students, regardless of their initial level of intelligence, fall far short of realizing their creative potential in mathematics;

(2) that fostering creative problem solving ability is one of the important goals of school mathematics;

(3) that creative problem solving can be regarded as thinking processes involving specific mental operations; and

(4) that creative problem solving can be learned and utilized by students as heuristic methods of thinking and tackling mathematical problem-situations.

DELIMITATIONS OF THE STUDY

The study is delimited to forty-one students drawn from two grade eight classes of one Edmonton Separate School. Grade eight Motion Geometry was selected as the subject matter domain upon which two different instructional units were constructed for the two different instructional methods. The units covered the first nineteen instructional objectives of the Junior High Mathematics Program: Geometry (Level Eight) (Edmonton Separate School Board, 1975). The experiment was conducted over a period of six weeks.

LIMITATIONS OF THE STUDY

The major limitations of the study reside in the small size of the sample (20 students in the experimental class; 21 in the control class), the specific content area selected (viz. motion geometry) and its limited scope, and the involvement of only a single teacher in a single school. Consequently, despite the fact that the two classes utilized were relatively heterogenous in composition[1], there is a limit to the extent to which the findings of this study may be generalized.

Furthermore, it should be noted that this is to date the first attempt to operationalize Boychuk's (1974) model of creative

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[1]St. Edmund Separate School does not employ special grouping procedures in allocating students to classes.

problem solving in the area of motion geometry. Specific creative problems had to be constructed as test items, and specific procedures for scoring designed for those items. While attempts have been made to validate the testing instruments and statistical methods employed to assess the reliabilities of each instrument, there is a need to interpret the results of the study with some caution.

SIGNIFICANCE OF THE STUDY

The study focuses on the feasibility of operationalizing Boychuk's (1974) psychological model of creative problem solving in school mathematics using junior high motion geometry as the subject matter domain. Since no theoretical model is really meaningful unless it can be translated into practical instruction at the classroom level, the outcomes of this study will essentially serve as a further validation of Boychuk's theoretical model.

In order to determine the merits of the instructional method derived from the psychological model, a control group was used and as many aspects as possible of the students' background investigated. The results of the study will therefore throw some light on the roles of instructional methods, subject content, and teacher classroom behavior in promoting creative problem solving ability in school mathematics.

CHAPTER II

REVIEW OF THE LITERATURE

INTRODUCTION

The present study is concerned with creative problem solving in the junior high mathematics curriculum. Consequently, the psychological theories underlying problem solving in general, and creative problem solving in mathematics in particular, will have to be examined to justify the utilization of Boychuk's (1974) psychological model. Furthermore, one of our major objectives is to translate Boychuk's theory into classroom instruction that would enhance students' mathematical creativity. Relevant instructional theories are hence needed to guide our development of creative instructional method and materials. In this regard, research studies on various methods of stimulating individual's creativity deserve to be reviewed. Though evaluation of the instructional outcomes of these various creative teaching methods requires special scoring procedures which involve some controversial issues, our considerations of this evaluating aspect will be given in Chapter VI where problems related to measuring instruments and scoring schemes are discussed. This chapter, therefore, deals with the related aspects of psychological theories, instructional theories and methods stimulating creativity.

PSYCHOLOGICAL THEORIES

The Role of Problem Solving in Mathematics Education

The mathematics enterprise, in its most general sense, consists of the solution of mathematical problems. In mathematics textbooks, the collection of problem exercises at the end of each chapter has traditionally served three purposes: (1) to illustrate the text materials, (2) to provide practise materials for students (Dilworth, 1966: 91), and (3) to assess students' understanding of the related concepts or skills (Jacobson, 1966: 102). More modern textbooks, however, are now deliberately including problems with new terms and symbols which seem unrelated to the experience and background of the students. Designed to prompt students to explore new ways of utilizing learned concepts and skills, such problems are indicative of a fundamental change in the perceived role for problem solving in mathematics education.

Over the last decade or so, there has been a growing awareness of the importance of problem solving in mathematics learning, as seen by the many studies undertaken in this area (Kilpatrick, 1969). Fundamentally, underlying these studies is the "realization that mathematics is not something which is passively learned, but is something which people do", and "specifically, mathematics at all levels, is chiefly concerned with problem solving" (Dilworth, 1966: 91). Furthermore,

> It has already been observed that the principal objectives of the new mathematics curricula are to give students deeper understanding of the basic

> mathematical concepts and to stimulate them to do creative and independent thinking with these concepts. Accordingly, it must be the general purpose of the problem sets to implement these objectives (Dilworth, 1966: 92).

Likewise, the Cambridge Conference on School Mathematics (1963: 28) claimed that "the composition of problem sequences is one of the largest and one of the most urgent tasks in curricular development."

The "active" characteristic of problem solving has been deemed important. As Fine (1966: 98) argues, "it is difficult to imagine what the teaching of mathematics would be like without problems", and "student activity is the most important ingredient of the learning process, and problem solving is the most common and effective form of activity." Likewise, Jacobson (1966: 103) welcomes this personal involvement aspect of learning in problem solving.

> Any mathematical activity that causes the student to become an <u>active participant in the proceedings</u> is a problem activity, and a good curriculum employs such activities throughout.
>
> The heuristic, or discovery method of teaching in the classroom involves such activities.

That there now exists wide recognition of the fundamental role of problem solving in psychological and instructional studies is demonstrated by the fact that the <u>Second International Congress on Mathematical Education</u> (Howson, 1973: 18) chose "Creativity, Investigation and Problem Solving" as one of the three topics for discussion. At a session of this conference, Biggs (1973: 221) stressed the importance of free investigation and problem solving as a means to encourage students to use their creative powers fully

in mathematics.

An adequate summing-up of the developing trends in mathematics education with regard to the role of problems in the development of mathematical activity in students has been provided by Rosenbloom (1966: 130):

> We regard problem solving as the basic mathematical activity. Since, in mathematical education, our first concern must be with what we want the student to do, we must focus our attention on this domain.
>
> Other mathematical activities such as generalization, abstraction, theory building, and concept formation are based on problem solving.

Creative Problem Solving in Mathematics

A student is confronted with a mathematical problem when he is given a mathematical situation which cannot be resolved by the immediate and direct application of mathematical processes known to him. We say that he is *thinking* when he is actively engaged in the task of arriving at some form of solution(s) to the problem. According to Klausmeier and Ripple (1971: 438), there are at least two directions of thinking in problem solving:

> One is toward acquiring a solution or closure to a problem for which there is a known or a generally accepted answer. *Convergent thinking*, *logical thinking*, *critical thinking*, and *reasoning* are terms used quite generally to describe this direction. Another is involved in seeking a new (at least to the thinker) or not generally accepted solution. This direction of thinking, called *divergent thinking* by Guilford, has been termed by others *creative thinking*, *imaginative thinking*, and *original thinking*.

The association of creativity with divergent thinking effectively began with Guilford's (1950, 1956, 1959, 1962, 1965, 1967) formulation of his three-dimensional model of the structure of human intelligence. The model is derived by a verbal process of distinguishing four kinds of <u>content</u>, six kinds of <u>product</u> and five kinds of <u>operation</u>, which give by their product 120 nameable intellectual abilities. The five operations are: cognition, memory, convergent production, divergent production and evaluation. He distinguishes between <u>convergent production</u> in which there is a single accepted answer to a problem, and <u>divergent production</u> in which a variety of answers is called for. Though Guilford does not equate divergent thinking with creativity, he suggests that there is a close relationship between abilities in this category and creative thinking, which is inseparable from problem solving.

> Most of the more obvious contributions to creative thinking are in the divergent production category. The factors of fluency, flexibility, originality and elaboration are in that category. It can be said that divergent production abilities are the most direct contributors to creativity (Guilford, 1965: 15).

> There is something creative about all genuine problem solving and creative production is typically carried out as a means to the end of solving some problems (Guilford, 1967: 314).

The cognitive processes which Guilford regards as belonging to the category of creativity are:

(1) <u>sensitivity</u>: an ability to generate many problems in response to a given situation;

(2) <u>fluency</u>: an ability to propose many ideas relevant to a given problem,

(3) flexibility: an ability to produce many different classes of ideas for a given problem,

(4) originality: an ability to give responses uniquely different from others,

(5) elaboration: an ability to state many details related to creative responses,

(6) redefinition: an ability to redefine the purposes of existing objects, techniques and facts in unconventional manner to facilitate problem solving.

(7) evaluation: an ability to perceive the adequacy of some solutions and to discard others.

Including both convergent and divergent production in his creative problem solving model, Guilford maintains that this is valid since both types of operations result in unique, distinct or unconventional solutions.

> One of the creative thinking abilities mentioned, redefinition, is classified with convergent thinking factors, a classification that may seem to be somewhat contradictory, but it is in the row for which the kind of product is that of transformations. Much creative effort is in the form of the transformation of something known into something else not previously known (Guilford, 1962: 162).

This model of creative problem solving in which divergent and convergent thinking are commingled is equally applicable in mathematical reasoning.

> Problem solving in mathematics often begins with divergent thinking. The mathematician seeks clues to the structure of the problem through a kind of intelligent guessing procedure. This trial and error procedure requires divergent production to generate new combinations and possibilities to be tried. The mathematician does

> not solve problems by random guesses, but he brings forth his store of previous knowledge to generate attempts based upon cognizance of the procedures of application of mathematical models and the careful analysis of the problem at hand.
>
> The outcome of this divergent thinking phase of problem solving is new information gathered from the trials. If the mathematician is able to put this information together in a related way to form a synthesis, then the problem solving proceeds to a phase of convergent thinking where the information is focused almost deductively towards a solution (Higgins, 1973: 45).

A similar succinct description of such creative problem solving processes has been given by Torrance (1962b: 40):

> The creative process seems to be quite well established and the process seems to be the same regardless of the activity. First, there is apparently the sensing of a need or deficiency, random exploration and clarification or pinning down of the problem. Then ensues a period of preparation, accompanied by reading, discussing, exploring, formulating many possible solutions and critically analyzing these solutions for advantages and disadvantages. Out of all these activities comes the birth of a new idea -- flash insight, illumination. Finally there is experimentation to evaluate the most promising solution and the selection of the idea.

This description bears affinity with the sequence of stages espoused by Hadamard (1954) to categorize the psychological processes involved in the mathematical field: (1) <u>preparation</u>, (2) <u>incubation</u>, (3) <u>illumination</u>, and (4) <u>verification</u>.

All these various characterizations of creative processes in solving mathematical problems lead to the conclusion that creative behavior in mathematical thinking begins with some sort of <u>sensitivity</u> to a problem situation. A mathematician is "aware of the possibility of alternative treatments" (Cambridge Conference on School Mathematics, 1963: 28). During this phase of divergent

thinking, a great variety of mathematical reasoning is involved, such as "extracting a mathematical concept from, or recognizing it in, a concrete situation, and then 'guessing' in many forms, anticipating the result, and anticipating the great lines of the proof before the details are filled in" (Polya, 1966: 124). Such guessing or conjecturing may involve inductive reasoning, generalization or arguments from analogy (Polya, 1962, 1965, 1966, 1971, 1973). The outcome may be a fluency of ideas. Moreover, as Boychuk (1974: 43) observes, Polya's suggestions of creating analogous or simpler problems, and of induction call for a rearrangement, a transformation or a redefinition of elements relevant to the problem. The concept of flexibility of ideas is certainly implied in Polya's problem solving strategies. It must be stressed however that ultimately, all ideas generated have to be evaluated and mathematically verified.

In her attempt towards a synthesis of this trend of research in mathematical creativity, Boychuk (1974) has postulated a creative problem solving model applicable in school mathematics instruction. Creativity is defined in this model in terms of four processes: two divergent processes (conjecturing, sensitivity) and two convergent processes (redefinition, verification).

Boychuk's Creative Problem Solving Model

In order to defend the premise that mathematical creativity can be defined validly and meaningfully in terms of problem solving processes, Boychuk (1974) compared several conceptual analyses of the creative thinking process and of the problem solving process.

Dewey (1910), for example, advocated a five-step problem solving model: (1) a difficulty is felt, (2) the difficulty is located and defined, (3) possible solutions are generated, (4) consequences are considered, and (5) a solution is accepted. Wallas (1945) and Hadamard (1954) proposed a four-stage model: (1) preparation, or the gathering of information, (2) incubation, or unconscious manipulation, (3) illumination, or the emergence of solutions, and (4) verification, or the testing of solutions. Rossman (1964) postulated a seven-step model: (1) need or difficulty observed, (2) problem formulated, (3) available information surveyed, (4) solutions formulated, (5) solutions critically examined, (6) new ideas formulated, and (7) new ideas tested and accepted. All the processes of these lists, with the exception of "incubation" appear to involve both divergent and convergent productions. Guilford (1967) suggests that incubation involves transformation of information resulting from induced interactions among stored products of information in memory.

Boychuk further examined Guilford's (1967: 315) sequential and operational model of problem solving schematized in the form of a flow chart to permit multiple feedback options. This model emphasizes the role of both divergent and convergent productions in generating solutions to problems. Extensive discussion was then carried out to interpret Polya's (1962, 1965, 1971) four-step problem solving strategies (understanding the problem, devising a plan, carrying out the plan, and looking back) in terms of Guilford's model (Boychuk, 1974: 39-45).

The following processes, two divergent (conjecturing, sensitivity) and two convergent (redefinition, verification) were identified as important component skills in creative problem solving applicable in mathematical context:

> (1) Conjecturing. Conjecturing is a divergent process which refers to the generation of hypotheses, of relationships, in response to a given source of data. The individual may conjecture by making naive guesses or by making deductive guesses.
>
> (2) Sensitivity. Sensitivity is a divergent process which includes the ability to perceive deficiencies and errors, shortcomings or inadequacies in a given situation, and the ability to see possibilities in a given situation; possibilities that lead to further questions.
>
> (3) Redefinition. Redefinition is a convergent process which includes the reassociation and the recombination of previously unassociated elements of knowledge to result in new combinations, and the discarding of previously adequate approaches in order to facilitate the perception of and the solution to a problem.
>
> (4) Verification. Verification is a convergent process which refers to the justification of a statement or relationship in four ways: by testing with specific examples, by establishing a rationale of assumptions, by producing suggestions by which a statement may be tested, or by formulating deductive proof (Boychuk, 1974: 12).

This model was developed to determine the extent to which junior high school students can react creatively to geometrical problems. After establishing specific guidelines for the formulation of appropriate problem-situations which reflect each of the four processes, two problems reflecting each process were constructed. The eight creative problems were then administered to forty-two grade nine students from four junior high schools in the Edmonton

Public School System, and written responses, oral elaboration and comments elicited from the students. Factor analysis of the data isolated six independent factors, three of which seem to indicate the processes of conjecturing, redefinition and verification, whereas the two sensitivity problems loaded on two other independent factors. As a whole, Boychuk's findings did not discredit her theoretical model.

Though the four processes are not regarded as exhaustive of all cognitive processes involved in creative problem solving in mathematical situations, they nevertheless provide a workable model for instructional research. Undoubtedly, creative problem solving is a rather complex human phenomenon which may involve many dimensions with a variety of mental operations. It is necessary therefore to define the universe of creative problem solving abilities in manageable terms (Treffinger and Poggio, 1972: 255). Boychuk's (1974) model conceptualized creative problem solving processes in mathematical thinking in terms of sequences of component mental operations, i.e. sensitivity, redefinition, conjecturing and verification. Such a theoretical model is meant to be <u>descriptive</u> of general features of creative thinking phenomena. The advantages of such a model have been aptly appraised by Messick (1973: 282):

> These models emphasize both the distinctiveness of the component processes and the sequential nature of their combination in achieving the final solutions or creative products. This suggests ... that overall aspects of the total process (and possibly its major phases) should be assessed directly to gauge relative effectiveness in combining the appropriate components in task performance.

In sum, Boychuk's creativity model in its essential outlines was deemed to be an appropriate theoretical framework for this study, although as we will show in the next chapter, modifications are needed to adapt her theoretical model to the domain of our study.

INSTRUCTIONAL THEORIES

Why Instructional Theory

The discussion above has indicated that Boychuk's (1974) creative problem solving model, as well as many other similar types of theoretical models, could function as a conceptual framework for research and investigation in creative teaching. According to Nuthall and Snook (1973: 47), there is a general tendency for those involved in research on teaching to construct descriptive models or interpretive theoretical frameworks of teacher-pupil relationships and interactions which are then used to synthesize and co-ordinate in a single structure those elements of observation and research findings.

The present study is based on Boychuk's theoretical model. However, it should be noted that no assumption has been made to claim that this is the only valid representation of current knowledge about mathematical creativity. As Nuthall and Snook (1973: 49-50) have indicated, "there are few logical or empirical connections between the models", and there is no sense "in which empirical evidence can be used to prove the validity of one model or demonstrate its superiority over another model." Indeed, recent investigations on the

historical development and the philosophical foundations of physical science have pointed out that one of the major functions of scientific models is to persuade and foster conviction that one way of looking at and interpreting physical phenomena is better and more fruitful than any alternative views (Kuhn, 1970, 1974). Consequently, it may be agreed that each model of teaching is fundamentally a claim about how teaching ought to be understood and interpreted.

Boychuk has reviewed relevant research literature on mathematical creativity and demonstrated the feasibility of analysing junior high students' responses to creative mathematical situations in terms of four problem solving processes. Her theoretical model seems to provide an appropriate guideline for assimilating and analysing information about creative behavior in school mathematics. Two major premises underlie this study: first, that creative problem solving abilities do exist as part of the developable intellectual capacity of junior high students, and second, that fostering creative thinking is one of the major functions of mathematics education. The crucial task then, consists of translating Boychuk's model into classroom instruction with the aim of enhancing mathematical creativity of junior high students in terms of the four processes of conjecturing, sensitivity, redefinition and verification.

In order to operationalize Boychuk's model at the classroom level, it is necessary to call upon the guidance of some relevant <u>instructional theory</u> or theories. There is first of all the need to heed Merrill and Wood's (1974: 79) serious criticism of instructional

researchers for failing to describe their treatments (instructions) adequately in terms of their theories.

> One of the difficulties in reviewing existing research literature, is the fact that when complex instructional strategies are involved, it is almost impossible to determine how the strategy was constructed.

Psychological models in research on mathematics learning usually do not prescribe practical principles concerning the most effective and efficient way of achieving instructional objectives implied by these models. On the other hand, "a theory of instruction, in short, is concerned with how what one wishes to teach can best be learned, with improving rather than describing learning" (Bruner, 1966a: 40). Johnson (1969: 123-124) has defined a model of instruction in a comprehensive statement designed to encompass all possible instructional situations:

> It must serve for both training and instruction, for all domains of learning outcomes, for both academic and non-academic content, for divergent as well as convergent learning, for all ages, abilities, and backgrounds of "instructees", for large and small groups as well as single individuals, for situations with and without teachers, for teachers of varying competence and personality, for programs, computers, simulation games, and responsive environments, and for all kinds of communities and every degree of availability of methods and equipment.

This useful definition recognized that there are four major components involved in any classroom instruction: (1) the content, (2) the instruction, (3) the teacher, and (4) the learner (DeVault and Kriewall, 1970). In this study, junior high motion Geometry constitutes the content domain. In addition, the assumptions that

mathematical creativity is normally distributed among students who would be the subjects under investigation, and that any teacher can effectively enhance students' creativity through appropriate instruction, are accepted as reasonable and justified in the context of the experiment. Only the "instruction" component therefore requires careful investigation for this study.

The Two Functions of Instruction

From their review of the literature on instructional research, Romberg and DeVault (1967: 100) have concluded that instructional tasks, instructional materials and organizational context of instruction are the three major aspects of instruction which have received most attention. They regard "instructional tasks" as those things a teacher does to facilitate learning. DeVault and Kriewall (1969: 83), however classify classroom management policies, instructional methods employed and kinds of materials used as the three basic instructional input variables. If the instructional materials or content can be taken for granted, then the other two aspects of instruction -- instructional tasks and organizational context, or instructional methods and management policies -- can be described in terms of the two basic functions of instruction identified by Johnson (1969: 125):

(1) Display: to convey meaning (content) with which the students can interact.

(2) Control: to regulate the interaction.

The instructional tasks or methods imply the "display" of relevant instructional materials through some way that students can learn effectively. On the other hand, teachers also "use controlling functions principally to tell children what it is they are to do; what questions to answer, what pages to read, what problems to solve, and so forth, as well as the sequence in which these activities are to be performed" (Hudgins, 1971: 81). Thus the major function of organizational context or management policies appears to be that of "control". In practice, however, these two functions are frequently inextricably intertwined.

> A skillful teacher may control the behavior of a child with a well-timed, well chosen question as completely as if she took direct management action to exercise such control. Similarly, skill in management is necessary to organize subgroups which in turn may be a prerequisite condition for certain kinds of teaching to occur (Hudgins, 1971: 16).

The display and control functions of instruction are also quite explicit in the Task Variable Taxonomy proposed by other researchers (Merrill and Boutwell, 1973; Merrill and Wood, 1974). The Taxonomy enables instructional strategies designed to promote higher cognitive behavior to be discussed along two dimensions each containing four variables: (1) the content oriented variables (content type, content mode, content representation and mathemagenic prompting), and (2) the learner oriented variables (response mode, response representation, mathemagenic feedback and response conditions). For each of those variables, several principal parameters have been suggested. Clearly, the type, mode and representation of the content are "display"-type variables, while the

mathemagenic prompting and the four learner oriented variables are "control"-type variables. Furthermore, according to the framework of this *Taxonomy*, different instructional methods or strategies can be constructed by forming various combinations and sequencing of those variables with their respective parameters. In principle, this means that different instructional methods can be designed through different manipulation of the *display* and *control* functions of instruction.

In summary, the various instructional analyses discussed in this chapter imply that an adequate instructional strategy has two fundamental dimensions: (1) the subject content to be displayed, and (2) the criteria which regulate the learners' behavior. Thus, in order to describe adequately in unambiguous terms the instructional strategies which were derived from Boychuk's model and which constitute the experimental treatments of our study, this two-dimensional classification was selected as the instructional model.

METHODS OF STIMULATING CREATIVITY

Stimulating General Creativity

The bulk of studies on creativity are concerned more with the description and measurement of creative abilities than with the promotion of creativity. Two of the few earliest efforts at stimulating and facilitating general creative problem solving abilities, however, were centered on the strategies of *brainstorming* and *synectics*. Osborn (1948, 1953) and Parnes & Meadow (1959, 1960,

1963) proposed brainstorming as a creative strategy requiring groups to produce large quantities of ideas under conditions which suspend criticism and evaluation. Experimentally, their use of brainstorming was successful in facilitating both the quantity and quality of ideas produced through individual and group thinking. Some of these findings, however, have been disputed by a number of other researchers (Taylor, Berry and Block, 1958; Dunnette, Campbell and Jaastad, 1963). Like Osborn's "brainstorming" thinkers, Gordon's (1956, 1961) "synectics" thinkers defer judgement, encourage wild ideas and utilize specific metaphor-based procedures for idea-finding. Synectics was initially designed as a system of training for industrial inventors. Consequently, very highly trained leaders are needed to act as key components in guiding "formal" synectics sessions.

Though the somewhat conflicting findings about the effectiveness of brainstorming raise difficulties for determining under what conditions this training method is most appropriate, it can be argued that this approach of producing large numbers of ideas is a good starting point for creative thinking. Synectics, on the other hand, stresses the emotional and irrational components of creative thinking. Such an emphasis seems directly antithetical to the teaching of mathematical creativity which involves rational and objective reasoning.

Covington and Crutchfield (1965) have attacked the problem of creative training by constructing special auto-instructional programs comprised of detective and mystery study materials. They found that elementary school children using the programs were

markedly superior to control children on problem solving and creativity measures. Though these original findings could not be replicated by Treffinger and Ripple (1968), Barron (1965: 108) nevertheless observes that,

> A special virtue of the Crutchfield and Covington demonstration is that it was made within the school system itself and resulted in the development of techniques that can be incorporated into existing curricula.

Furthermore, the emphasis on both convergent and divergent thinking in solving problems of the auto-instructional programs (Covington, Crutchfield and Davies, 1966; Covington, 1968) comes very close to creative problem solving in school mathematics.

Another direct approach to the development of creativity is Davis and Houtman's (1968) four-method strategy: (1) Part-Changing Method, (2) Checkboard Method, (3) Checklist for Finding Ideas, and (4) Find Something Similar. Each of the four methods is a specific skill which encourages and guides students to come up with many new ideas. Substantial evidence has been accumulated to demonstrate that creative abilities of school children can be improved significantly through these four idea-generating methods (Feldhusen, Treffinger and Bahlke, 1970).

It is noted that most of these theories of training seem to start from the "operations" categories of Guilford's structure-of-intellect model mentioned previously. The common underlying assumption is that voluminous productivity is the rule, not the exception, among probably all creative individuals (McGannon, 1972: 8). In addition, guidelines for improving creative abilities are necessarily as follows:

(1) Encourage divergent production in many media.
(2) Reward creative efforts.
(3) Foster a creative personality (Klausmeier and Ripple, 1971: 469).

Teaching for Mathematical Creativity

Discovery Method. "Discovery Method" is probably the earliest theory-based pedagogical technique employed by many mathematics educators to foster creative problem solving abilities in school children. Those who advocate teaching by discovery, such as Bruner (1960, 1961, 1963, 1966a, 1966b), Davis (1964, 1966) and Polya (1962, 1965, 1966, 1971, 1973) claim that the virtue of discovery learning lies in the following consequences:

(1) Students are motivated to learn mathematics.
(2) They will understand what they learn.
(3) They will learn to think.
(4) They will become more creative (Brown, 1971: 233).

Others like Ausubel (1961, 1964, 1968) and Cronbach (1966), however, have serious reservations about the rationale and effects of the discovery method. While the empirical evidence is inconclusive with respect to the merits and disadvantages of this approach (Cooney, Davis and Henderson, 1965: 170), a brief discussion of some of the discovery strategies may shed light on the design of instructional strategies for creative learning in school mathematics.

There appears to be five different kinds of discovery learning approaches (Biggs, 1972: 217-218):

(1) Fortuitous Discovery: This type of discovery is always initiated by the learner, being unplanned and not in any way teacher-directed. It is perhaps the most highly motivated type of discovery

a learner can achieve.

(2) Free and Exploratory Discovery: Learning materials are provided with minimum structure. The typical situation is Dienes' (1967) stage of preliminary games, where children are given freedom to experiment with the materials in any conceivable way.

(3) Guided Discovery: Children are guided in their exploration of given learning materials, through the help of sequences of questions from the teacher.

(4) Directed Discovery: This is a more structured learning situation than is guided discovery, where questions and directions are more flexible.

(5) Programmed Discovery: Here, instead of direct instructions from the teacher, children are given well prepared instruction sheets or cards to follow.

In short, at one extreme of the discovery strategies continuum, the situation is so open that discoveries made by the children may be new to even the teachers. It may be argued that such an instructional form is too demanding for the average classroom teacher, since the success of free and unstructured discovery probably requires creativity and high competence in subject matter on the part of the teacher. Consequently, it would be unrealistic to advocate the "fortuitous" and "free-exploratory" strategies of discovery in the ordinary mathematics classroom (Varga, 1971: 28). At the other extreme, the teacher determines the precise mathematical facts to be discovered, and leads the children to these facts through structured sequences of questions. Success is more or

less guaranteed by careful planning and implementation. It can be argued therefore that the guided, directed and programmed discovery methods are more practical for ordinary classroom usage and easier incorporation into the normal teaching activity.

Generally, the discovery approach is viewed as a problem solving model employed by most of the users and inventors of mathematics. What mathematicians do first, when confronted with a mathematical situation, is

> to explore a situation, a state of affairs, concrete or abstract, which by its presence constitutes a challenge to their powers of discernment and invention. In the course of this exploration, ideas are clarified, some factors are recognized as important, while others are discarded. A scheme begins to take shape, whereby one or more problems are formulated and attempts are then made to solve them. The solution leads to establishing properties or accepting others as hypotheses or conjectures. Gradually a system of relations is built up, which must then be tested with all available means: intuitive models, logical deduction, counter-examples (Servais, 1971: 242).

Many studies which have explored the feasibility of fostering creativity in school mathematics have pursued the discovery model of mathematical thinking which involves such cognitive processes as sensitivity, conjecturing, hypothesizing, evaluation and verification. Clark (1967) demonstrated that through creative teaching methods concerning story problems, creative talents can be used effectively while students are acquiring mathematical knowledge. The control group was taught in a traditional manner, where the "one correct way" to solve problems was the emphasized feature. Meanwhile, the experimental group was given no explanations, but the students were expected to explore the problems on their own. A

considerable amount of class time was devoted to group discussion and evaluation of various solutions suggested by students. Although the control group solved twice as many story problems as the experimental group during the study, the latter showed greater gains in problem solving ability as measured by the net gain on pre and post tests.

Another creative classroom environment was designed by Buckeye (1968) in the following way. Creative thinking of students was developed through encouraging and respecting students' questions or imaginative and unusual ideas, allowing opportunities for practice and experimentation without evaluation, and assigning challenging and enrichment problems which were appropriate to students' resources. The conventional lecture method was employed for the control group. Significant increases in creative ability were obtained for the experimental group in comparison with the control group.

There is, however, no firm consensus on the feasibility of creative instruction. Meyer (1970) and Borgen (1971), for example, observed no significant differences in either general creativity or mathematical creativity in students exposed to special creative teaching. Nevertheless, despite the non-uniformity of results, there are remarkable similarities among different creative instruction through discovery approaches. These may be summarized in the following guidelines:

(1) Some mathematical problems or situations are displayed to the students, who are then asked to explore the problems or

situations under the guidance of certain directions.

(2) Students are encouraged to investigate the problems in as many conceivable ways as possible (Higgins, 1973: 89), and small group or class discussion is normally essential (Biggs, 1972: 218).

(3) Some of the heuristic means of discovery and invention, such as conjecturing, transformation, redefinition, analogy, induction, and plausible reasoning (Polya, 1962, 1965, 1966, 1971, 1973) are invoked to resolve the problems.

(4) In general, most of the discovery approaches emphasize explicitly or implicitly the desirability of fluency, flexibility and originality of ideas in any discovery learning situation in school mathematics.

Mathematizing Mode. One of the major difficulties to date in all attempts to resolve the learning-by-discovery controversy is a fundamental lack of agreement concerning a precise definition of the discovery method (Dessart and Frandsen, 1973: 1182). The "mathematizing mode" (Johnston, 1968; Naciuk, 1969; Taylor-Pearce, 1969; Tobert, 1969; Sigurdson, 1970; Sigurdson and Johnston, 1970; Tschofen, 1973) represents one of the attempts to formalize the discovery method as a sequence of four instructional stages (Johnston, 1968: 61-62):

(1) Introduction of the Activity: Free exploration of a given problem situation by students.

(2) Brainstorming Session: Encouraging ideas and suggestions from students without evaluation.

(3) Seminar Type Discussion: Indicating questions and

problems to enable students to test out their ideas and hypotheses.

(4) Summary: Summing up of the precise mathematical principles involved in the preceding stages by students.

These four stages have been refined by Tschofen (1973: 119) to six stages: introduction, exploration, hypothesizing, evaluation of hypotheses, summary, and practice.

Essentially, the mathematizing mode emphasizes the processes of discovery and invention in mathematics learning.

> If mathematics is created by hypothesizing, evaluating, and rejecting or accepting, and many mathematicians would agree that it is, then practice in these activities can only improve the students' ability to handle mathematics creatively (Sigurdson and Johnston, 1970: 133).

However, Taylor-Pearce (1969) found that for each divergent thinking criterion (fluency, flexibility, originality, and total response), the treatment effects of the expository method were significantly superior to the treatment effects of the mathematizing method. These findings should probably have been expected because of the content-loaded nature of Taylor-Pearce's creativity tests. For example, the following questions (in paraphrased form) were given (Taylor-Pearce, 1969: 50-51):

-- Write ten true statements about an integer which is divisible by 39.

-- Think of five possible values of x in the sequence: 25, 625, x.

-- Think out five practical ways of representing a mapping.

-- Find three sets of integers (m, n, q) satisfying the equation: $m^2 + n^2 = q^2$.

-- Write down seven sets of integers (m, n, q) satisfying the equation: $m^3 + n^3 = q^3$.

-- Think out five possible functions f(x) in the sequence: (x^2+2x+1), (x^2+6x+9), $f(x)$.

-- Write down ten true statements about the quadratic function $y = x^2-5x+6$, to illustrate the various mathematical quality of the function.

Clearly, knowledge about number properties, series, mappings, functions, operations, and quadratic equations in particular, are prerequisites for creative productions measured by the tests. On the other hand, it is well-known that expository teaching can effectively and efficiently present a rich body of highly related facts, concepts and principles which the students can learn and transfer (DeCecco, 1968: 468). The better performance of Taylor-Pearce's expository classes on the creative tests is not surprising. Similarly, this would seem to explain why in Tobert's (1969) study of these same classes, expository classes out-performed the mathematizing classes on achievement tests.

In sum, it should be pointed out that the assessment of mathematical creativity as an outcome or product of learning necessarily requires the students to utilize mathematical knowledge, concepts, rules, principles, and skills learned. Consequently, "it must be either assumed, or preferably shown, that students have in fact learned relevant prerequisite information and skills, before the assessment of 'originality' is undertaken" (Gagne and Briggs, 1974: 172). This study will be designed to take the latter advice into account.

SUMMARY

This chapter attempted to show that solving mathematical problems is the basic mathematical activity of almost all learning in school mathematics. Furthermore, solving mathematical problems inevitably calls upon one or both of the two kinds of thinking, (1) Convergent, and (2) Divergent. Though most of the theorists regard creative problem solving as involving more or less exclusively divergent thinking, Boychuk (1974) summarized relevant theories in this area and postulated a four-process model, including two convergent processes (redefinition and verification), and two divergent processes (sensitivity and conjecturing). This theoretical model, validated by Boychuk through a factor analytical method in junior high school mathematics, is deemed appropriate as the theoretical framework for our study.

The related research literature also indicated the need for appropriate instructional theory to help translate the theoretical model into classroom instruction. Johnson's (1969) theory of instruction which analyses instruction in terms of two basic functions, (1) display, and (2) control, was discussed as being practical and useful for guiding the operationalization of Boychuk's psychological model.

Finally, some instructional approaches which emphasize explicitly or implicitly the fluency, flexibility and originality of ideas as instructional goals, were analysed. In particular, the discovery method and the mathematizing mode of instruction in school mathematics were discussed in some detail.

CHAPTER III

THE CONCEPTUAL FRAMEWORK OF THE STUDY

INTRODUCTION

> If the classroom instructor is to plan instruction to encourage creative responses in his students, he will need to identify and circumscribe the classes of behavior which, when exhibited, will reflect creativity. Perhaps a consideration of the various kinds of cognitive operations which may be exhibited would be a useful approach. That is to say, the number and kinds of novel yet adaptable associations, identifications, classifications, hypotheses, and implications which can be generated from a given portion of subject matter would be utilized as criteria. In any event, the instructor must have some recognizable criteria to plan appropriate instruction and evaluate its effectiveness (Lembo, 1969: 167-168).

As the review of the literature has shown, there exists a number of controversies regarding the nature of the creative problem solving process and the instructional strategies that are optimally effective for increasing creative behavior in school mathematics. Lembo's above suggestions appear to identify four major dimensions involved in any discussion of creativity related to school instruction: (1) the <u>conceptual definition</u>, (2) the <u>operational definition</u>, (3) the related <u>domain of knowledge</u>, and (4) the <u>creative instruction</u>. These four dimensions seem to provide useful and well-defined guidelines for the development and evaluation of our study. In the following sections, we will show briefly the relevancy of these dimensions to the present study.

(1) Conceptual Definition of Creativity

Cognitive Operations postulated by various theorists, such as Guilford's category of "operations", have usually been heuristically or conceptually employed by different researchers to describe creative thinking phenomena. According to Treffinger, Renzulli and Feldhusen (1971: 105), this is the dimension of theoretical description of creativity. Conceptually, creative thinking has been defined in terms of some of the following mental operations or processes: sensitivity, conjecturing, elaboration, transformation, redefinition, hypothesizing, evaluation and verification.[1]

(2) Operational Definition of Creativity

Creative problem solving processes can be meaningfully studied only by getting students to generate observable "classes of behaviors." Many researchers have operationally categorized these behaviors into fluency, flexibility and originality. The criterion problem (Treffinger, Renzulli and Feldhusen, 1971: 107) arises when there is an apparent logical or empirical discrepancy between our conceptual and operational definitions (Crockenberg, 1972: 40-43). Some researchers create difficulties by imposing a prevalent conceptual meaning of creativity on what was meant to be a narrow operational definition for measurement, without specifying the relationships explicitly and precisely.

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[1]See Chapter II.

(3) Domain of Knowledge

Generally, attempts at improvement and assessment of creative thinking have tended to rely heavily on superficial types of problems, such as "How many uses can you think of for a brick?" It has been argued, however, that creative thinking should depend on knowledge of content matter as well as some sort of general creative ability (Johnson and Kidder, 1972). As our discussion on the "mathematizing mode" in Chapter II shows, there is a reasonable case for acknowledging the importance of subject matter domain in the study of creativity.

(4) Creative Instruction

Most researchers of creative thinking assume that creativity is one of the human cognitive abilities which can be developed through practice. They also claim to have empirically identified some of the major processes or intellectual skills that contribute most to creative behaviors. In effect, it seems justified to search for some "appropriate instruction" which can be used to train students in those processes and make them more "creative".

This chapter will therefore examine these four major dimensions of creative thinking as have been articulated in relevant literature.

CONCEPTUAL DEFINITION OF MATHEMATICAL CREATIVITY

Analysis of Boychuk's Model

Treffinger, Renzulli and Feldhusen (1971: 105) have pointed out that "there is no single, widely accepted theory of creativity which can serve to unify and direct our efforts at specifying an adequate assessment procedure." While not disagreeing with this fact, we have argued in the previous chapter that Boychuk's (1974) model can be considered an appropriate functional theoretical framework for the understanding and investigation of creative problem solving behavior in school mathematics. The relevant details of this model will now be described.

Basically, Boychuk (1974: 52-93) defines mathematical creativity in terms of four processes: Sensitivity, Redefinition, Conjecturing and Verification. The solution of creative mathematical problems invariably involves some or all of these four processes.

(1) Sensitivity. This is a divergent process which enables an individual to sense or become aware of the deficiencies, possible implications, or implicit shortcomings of a problem-situation. In other words, she/he is able to extrapolate beyond the obvious. While sensitivity necessarily implies in-depth understanding of a given problem, understanding in the general sense can occur at a level without the additional awareness of possible embedded difficulties, facts, alternatives, or implications.

(2) Redefinition. This is a convergent process which

involves the abstraction, transformation, decomposition and reassociation of elements of a given problem into new combinations to facilitate solutions. Redefinition includes the ability to overcome mental fixation, enabling the problem-solver to associate previously unrelated elements while discarding familiar associations which obstruct the solution of the problem.

(3) Conjecturing. This is a divergent process consisting of making naive or deductive guesses about the solution to a given problem. New ideas or hypotheses are generated from information given or learned in response to a problem-situation. This process stimulates the problem-solver to generate ideas about conditions, consequences or implications possible in the given situation.

(4) Verification. This is a convergent process which can occur either through a naive way of testing or a more sophisticated method of deductive proof produced to justify a given result or generalization. Verification also implies the evaluation or testing of possible hypotheses, enabling the problem-solver to accept or reject conclusions made to a given collection of data.

These four creative problem solving processes can be classified into two categories: (1) sensitivity, redefinition, and conjecturing, and (2) verification. The first three processes are needed to search for solutions to problems, whereas the process of verification is to justify the solutions found or given. However, there is probably continual interplay between "verification" and "conjecturing" (Boychuk, 1974: 88), as well as "sensitivity" and "redefinition". While solutions can be arrived at through naive

or deductive guessing, mathematical justification requires actual verification by testing or proving.

According to Boychuk (1974: 89), the verification of a mathematical statement means (1) to prove or disprove its truth by producing examples or counter-examples, (2) to justify and explain it by appealing to accepted assumptions, or (3) to produce a logical proof deductively. However, the processes of sensitivity, conjecturing and redefinition are the underlying processes which produce the relevant examples or counter-examples, the related assumptions needed for the justification and explanation, and the sequence of ideas that build up the deductive proof. As a mathematician Morris (1966: 58-59) has argued, logical proof merely sanctions mathematical production which is the fruit of intuition, while on the other hand creative thinking in mathematics entails "thinking for oneself, which means guessing, conjecturing, blundering, trial and error, induction from concrete evidence and all the other diverse and often haphazard processes which enter into thinking."

In effect, verifying facts, hypotheses or generalizations in mathematics means the whole process of constructing formal or informal "proof". This process of verification can be characterized as the combination of two basic stages: (1) the construction of a hypothesis or generalization, and (2) the construction of a "proof" or "disproof" (Lovell, 1971: 66-67).

> In the construction of the hypothesis, the rules of logic are generally of little value, for it requires some new combination of the problem solver's knowledge and the data of the problem -- this is an attempt to "close a gap". It might be hypothesized, for example,

> that a certain relationship existed between some variables in a problem, or that a property observed in a finite number of instances can be extended to a wider class of elements. To the hypothesis and the data of the problem the rules of logic may be applied in an attempt to arrive at a proof or disproof.

Similarly, Strike (1975: 477, 471) also makes a distinction between "the processes of the formulation of an hypothesis and its confirmation." Through a logical analysis, he showed that the "act of discovery" involves two basic types of cognitive skills: (1) heuristic skills: skills pertaining to the formulation of hypotheses, and (2) epistemic skills: skills pertaining to their verification. It appears then that sensitivity, redefinition and conjecturing are undoubtedly some of the important processes related to the heuristic skills of discovery, as well as creative thinking.

Indeed, Kline (1966: 57) points out that "conjectures must precede proofs." He also argues that the creative process, constituting "by far the largest and most difficult part of mathematical activity, is not contained in the axiomatic approach" which requires rigorous deduction from stated definitions and mathematically adequate assumptions or axioms, to justify every mathematical fact or conclusion.

The Three-Process Model

In the previous section, we have shown that the creative process of "verification" consists of two dimensions: (a) the construction of hypotheses, and (b) the construction of proof (or disproof). The former involves the processes of sensitivity,

redefinition and conjecturing and thus can be conveniently subsumed under these three creative processes. Consequently, Boychuk's creative problem solving model can be restructured to contain these four creative components, categorized into two independent dimensions: (1) sensitivity, redefinition, and conjecturing, and (2) construction of proof. The first dimension consists of three informal cognitive abilities which are not contingent upon a particular set of mathematical skills. Therefore, it is justifiable to assume that students can respond meaningfully to creative problem-situations calling for the functioning of these three processes if the situations are within students' resources. The second dimension, the construction of proof, however, would involve the mathematical concept of "proof" and various methods of proof. Discussion of mathematical proof inevitably includes such notions as undefined terms, definitions, axioms, theorems, deduction, and different methods of proof, e.g. direct proof, contrapositive proof, proof by enumeration, proof by existence, disproof by counter-example, and disproof by contradiction. Such notions and methods of proof require a fairly sophisticated level of mathematical competency. Since our objective is to investigate mathematical creativity of junior high students, it is necessary therefore to determine first whether these elements of proof construction form a part of the students' mathematical background.

Conventionally, "mathematical proof" in school mathematics is taught only in "geometry". According to the curriculum guide issued by the Alberta Department of Education (1975: 30-34), the

junior high mathematics program consists of the following geometry topics:

<u>Grade VII</u>:

1. Development of the ability to recognize and identify the following elements of plane Geometry, and extending knowledge of the interrelationships of these elements: point, line, plane, segment, ray, curve, closed curve, angle, triangle, other simple polygons, circle, interior and exterior regions.

<u>Grade VIII and IX</u>:

1. The measure and comparison of segments using British and metric units.
2. Measurement of angles.
3. Categorization of the types of angles formed by the intersection of coplanar lines.
4. The triangle, including classifications, similarity, perimeters, areas, and the unique property of the sum of interior angles.
5. The quadrilaterals, including classifications, perimeters and areas.
6. Classification of polygons.
7. Simple geometric constructions: bisectors of angles and segments, construction of parallels and perpendiculars, and construction of simple polygons.
8. The circumference and area of circles.
9. Applications of geometry to the solution of problems.

A careful survey of these instructional objectives and the recommended texts clearly indicates that the construction of rigorous mathematical proof is not a part of the junior high student's repertoire of mathematical experiences. Consequently, only the "construction of hypotheses" stage of the process of "verification" can be regarded as relevant to this study. It has also been shown that this stage can be meaningfully and validly described in terms of the other three creative problem solving processes. In sum, this

study conceptually defines creative problem solving in terms of the three processes of sensitivity, redefinition and conjecturing.

MATHEMATICAL CREATIVITY AS RULE GOVERNED BEHAVIORS

Boychuk's Problem Guidelines

Most researchers would probably agree that the solution of a mathematical problem is "typically a poor index of the processes used to arrive at that solution" (Kilpatrick, 1969: 160). It is important, therefore to study creative problem solving processes by eliciting observable sequences of behavior from the students through well designed problem-situations. In turn, this requires the construction or selection of appropriate problems which will evoke and facilitate the functioning of creative processes on the part of students.

Boychuk (1974) has suggested some broad guidelines for devising creative problem-situations.

(1) Sensitivity Problem-Situations: Situations which are designed to allow the student to exhibit puzzlement, to recognize shortcomings. These deficiencies however, should not be specifically called to attention by the instructions of the problem-situation (Boychuk, 1974: 59).

(2) Redefinition Problem-Situations: In such a situation, a "mental set" is first established by one of two means: (a) a pattern for solving a problem, organizing data, or constructing problems is established, or (b) objects or situations which are

commonplace and have specifically defined uses are employed in the structure of the problem-situation. Then the students will be called upon to perform one or more of the two tasks: (a) to combine two or more previously unassociated elements to produce a new combination which is the required solution, and (b) to solve a problem which is similar in appearance, format or situation to one just previously experienced, but is more appropriately solved using alternate procedures (Boychuk, 1974: 73-74).

(3) Conjecturing Problem-Situations: In this type of problem, a "thought-provoking" situation is first presented. Students are then required to produce many mathematical statements related to the given situation. A sample hypothesis is included as instruction to reflect the emphasis desired (Boychuk, 1974: 85-86).

In this investigator's opinion, there are three serious weaknesses in these guidelines. First, they are too general and ambiguous. What constitutes, for example, the kinds of problems which are likely to provoke puzzlement in the student, contain recognizable shortcomings or deficiencies, or simulate a "thought-provoking" situation? What pattern of solution or mental set is more likely to elicit the process of redefinition? Secondly, to be creative implies the flexibility in thinking, which enables a problem-solver to employ more than one process in tackling any given problem-situation. If "valid" problems are constructed closely in line with these guidelines, students are in effect "forced" to utilize the specified creative process to produce desirable solutions. This would be inconsistent with the very definition of

creativity. On the other hand, if the problems are not strictly "valid" -- that is, students are allowed to arrive at solutions via any one or more of the creative processes -- there would then be no need for separating the three problem-situations into distinctive categories. Finally, there seems to be some practical difficulty in constructing creative problem-situations which reflect one and only one of the processes for each category. As Boychuk (1974: 61) herself conceded,

> The original intention was to keep the processes as separate entities; however, in the interpretation of the ideas into practical problems, the separateness did not always seem possible. These problems are then designed to maximize the use of the sensitivity processes, but are not equivalent to the process.

Furthermore, she found high correlations between the three scores (fluency, variety and novelty)[2] for each question, as well as the existence of independent factors associated with each creative process. Boychuk (1974: 219) concluded that "the hypothetical processes of conjecturing, sensitivity, redefinition and verifying may be independent but that problems calling for only one process, and not another, cannot be constructed at a non-superficial level."

Rule Learning in Mathematics

Given the inherent inconsistency in the categorization of problem-situations in Boychuk's model, as well as the ambiguous

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[2]Boychuk's operational definition of creativity in terms of fluency, variety and novelty will be discussed in the following section.

and impractical nature of her guidelines, it seems desirable therefore to further analyse the creative process so as to obtain some common denominators which may avoid such difficulties. One useful theoretical framework in this regard appears to be the notion of "Rule Governed Behavior" (Scandura, 1966, 1968, 1970, 1971; Gagne, 1970; Gagne and Briggs, 1974).

According to Gagne and Briggs (1974: 43), a "rule" is an inferred capacity which enables the learner to respond to a class of stimulus situations with a class of relationships among classes of objects and events. Utilizing his favorite Set-Function Language (SFL), Scandura (1966, 1968, 1970) defines a rule as a set of stimulus domain (D) which determine a set of response range (R) through the application of an operation (O). In essentially similar terms, Merrill and Wood (1974: 22) states that "a rule is an ordered relation consisting of a set of domain concepts, an operation, and a set of range concepts." This relationship is illustrated in Figure 3.1.

Problem-solving is then described as a form of rule-governed behavior. When a learner is exposed to a problem-situation, she/he is required to identify the appropriate rule(s), combine available rules in new ways, or perhaps, even invent new rules to tackle the problem.

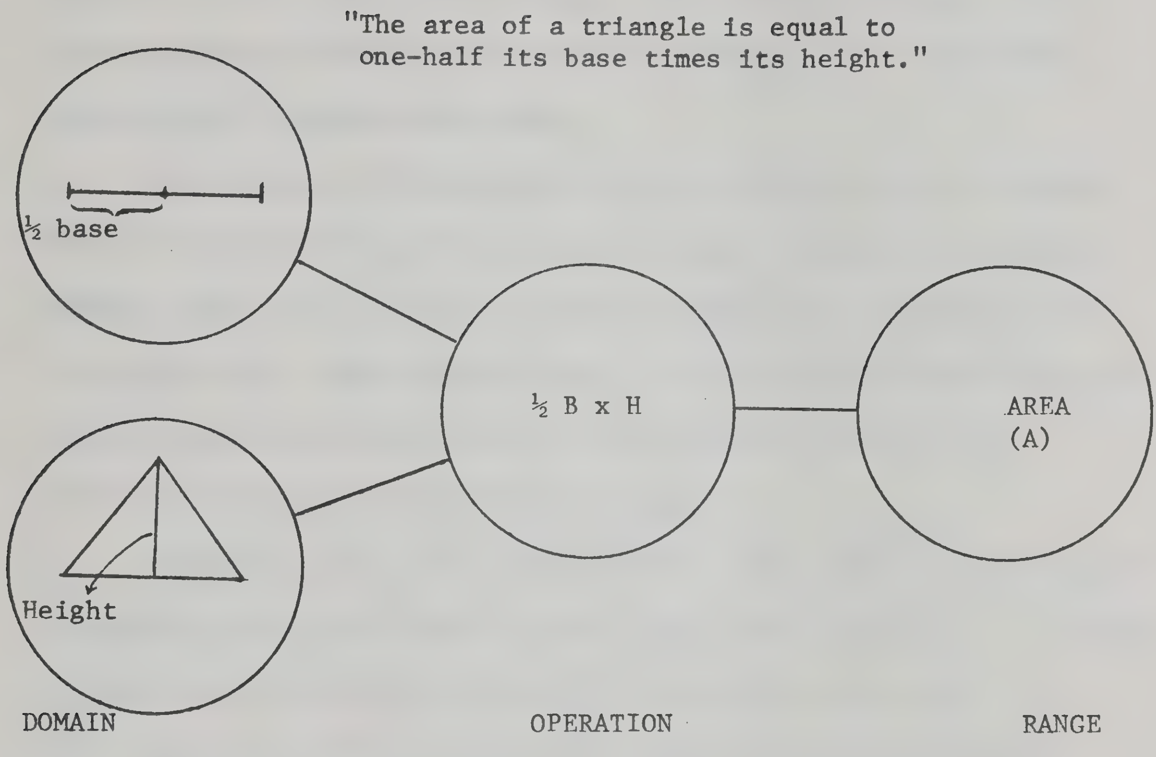


Figure 3.1 Sample rule expanded into domain, operation, and range concepts. (From M.D. Merrill and N.D. Wood, <u>Instructional Strategies: A Preliminary Taxonomy</u>, ERIC Information Analysis Center for Science, Mathematics and Environmental Education, Ohio, 1974: 23.)

> When a student works out the solution to a problem which represents real events, he is engaging in the behavior of thinking. There are, of course, many kinds of problems, and an even greater number of possible solutions to them. In attaining a workable solution to a problem, the student also achieves a new capability. He learns something which can be generalized to other problems having similar formal characteristics. This means he has acquired a new rule, or perhaps a new set of rules (Gagne and Briggs, 1974: 45).

Scandura, Gagne and Briggs also talk about "higher-order rules" which act on other rules. For instance, in the above example, the formula for finding the area of a triangle is a rule. A student

who is able to break up a parallelogram into two equal triangles applies the area rule to one triangle, and obtains the area of the parallelogram by doubling the area of the triangle, is essentially operating with higher-order rules.

The purpose of the present study, however, is not to assess the merits or validity of such a psychological theory of mathematics learning, but to utilize the language of domain (D), operation (O), and range (R) as common denominators to facilitate the construction of problem-situations reflecting the three creative processes of sensitivity, redefinition and conjecturing.

Scandura (1971: 36) firmly believes that "given suitable rules of combination, much of what normally goes under the rubric of creative behavior can be accounted for in terms of finite rule sets." In this respect, Merrill and Wood (1974: 29-33) give the following examples of creative problem solving situations described in terms of the three parameters of domain, operation and range:

> (1) Domain Finding: The student is given unencountered instances of some range concepts and the operation, and asked to find domain concepts with instances such that the operation will indeed produce the instance of the range given.
>
> (2) Operation and Range Finding: The student is given instances of the domain concepts and asked to find his own operation and to define his own range.
>
> (3) Domain and Operation Finding: This situation merely instructs the student to do something to produce the desired range, and the domain and operation are all left for his selection with only a given range label.

This appears to be a very meaningful way of describing the processes of creative problem solving in school mathematics. For

instance, the first type of problem solving can be regarded as a sensitivity-type of problem if the operation is specified in a broad and flexible manner so as to evoke students to sense a variety of possibilities. The second and third types are situations which seem to require conjecturing. It is worthwhile, therefore, to examine further implications of the notion of rule-governed behavior for investigating mathematical creativity.

Creative Problem Solving as Rule Learning

Consider first the eight possible types of problem-situations that may arise from combinations of the three parameters of domain (D), operation (O), and range (R).

PROBLEM-SITUATION	DOMAIN (D)	OPERATION (O)	RANGE (R)
1	v	v	v
2	?	?	?
3	v	v	?
4	v	?	v
5	?	v	v
6	v	?	?
7	?	v	?
8	?	?	v

v: Information given

?: Unknown to be found

TABLE 3.1 Eight Possible Types of Problem-Situations

Clearly, types (1) and (2) of Table 3.1 do not establish problem-situations, since the former simply displays a rule, whereas the latter is an empty, hence meaningless, situation. Type (3), however can be a creative situation if the O is given in a broad scope such that the student has to make use of many combinations or sequences of operations. Type (5) may be regarded as equivalent to type (3) since it can be restated by interchanging the D and R with the "inverse" operation O^{-1}. If such reversibility be embedded in the problem facing the student, a typical redefinition problem-situation (Boychuk, 1974: 68) is obtained. A creative situation is also found in type (4) if many permissible O's exist. Types (6) and (8) constitute the "operation and range finding" and "domain and operation finding" types of problem-situations of Merrill and Wood (1974: 29-33) discussed in the previous section. Lastly, type (7) can be a conjecturing problem-situation.

Traditional and Creative Problem-Situations

It will now be suggested that the three parameter sets (D, O, R) afford the basis for constructing two different problem-situation <u>structures</u> according to the following criterion questions:

(a) Are the given and the required parameter sets stated in broad or very specific terms?

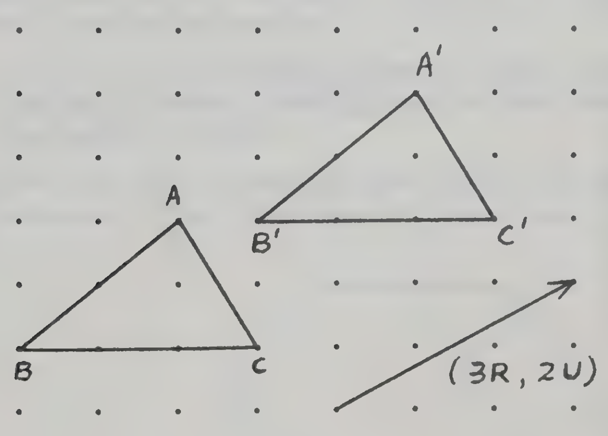
(b) Is only a single and specific correct solution set required, or a variety of permissible sets?

Different answers to these two questions give rise on the

one hand to the traditional structure of problem-situations, and on the other, to the creative structure.

(1) Traditional Structure. In problem-situations of this structure, two of the parameter sets are given in very precise terms, and the students required to obtain one and only one solution set for the third parameter. This is what Merrill and Boutwell (1973: 100) mean by a problem, "the presentation of a member of D set with instructions to the student to apply the operation, thus producing one and only one member of the R set."

It is necessary to stress that the emphasis lies not on which two of the three parameter sets are given, but on the characteristics of the given sets and the solution set to be found. For example, in the "motion geometry" situation below, D = {original $\triangle$ ABC}, O = {slide (3R,2U)}, and R = {image $\triangle$ A'B'C'}, three traditional problem-situations can be constructed:



(a) Find the image (R) of the $\triangle$ ABC (D) under the slide (O): (3R,2U).

(b) Find the single slide (O), which will map the $\triangle$ ABC (D) onto the $\triangle$ A'B'C' (R).

(c) Find the original $\triangle ABC$ (D) whose image is $\triangle A'B'C'$ (R) under the slide (O): (3R,2U).

In addition, traditional problems may ask for a specific solution set with more than one element. For example,

Find the real roots of the quadratic equation

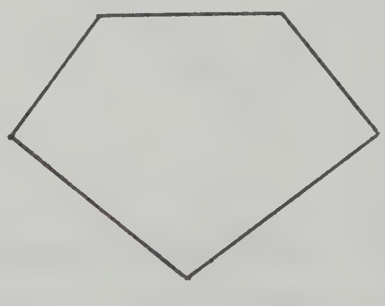
$$x^2 - 3x + 2 = 0$$

$$\text{Solution Set} = \{1, 2\}.$$

(2) Creative Structure. In problem-situations of this structure, one or two or the parameter sets are given in quite broad terms with many possible interpretations and implications embedded, and the students required to search for a variety of solutions or solution sets for the remaining parameter set(s). Flexibility is the key feature of creative problem-situations.

The most "open" creative problem-situations appear to be the conjecturing-type constructed by Boychuk (1974: 83):

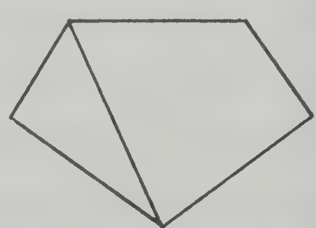
> You are given the following shape. Make as many conjectures as you can about the given shape. One example is the following:
>
> A series of pentagons cannot cover a flat surface without leaving gaps unless the pentagons overlap.
>
> You can use this statement and vary it to make your own conjectures. Then make some of your own.



For the present investigation, this sort of "open" question is deemed inappropriate, since it is too vague with no indication of the characteristics of the permissible solution sets (R and/or O). Responses such as "it reminds me of a barn, ... a water trough, a symbol of luck ..." were not accepted by Boychuk (1974: 121) because they are mathematically irrelevant and hence inappropriate. However, though the example given for this problem as an instructional hint may be seen to involve some topological properties, the statement "covering a flat surface without leaving gaps" cannot really be regarded as a kind of mathematical operation typically understood by junior high students. Moreover, the result of some "impossible" situation does not help to tell the student anything about the range that would satisfy the requirement of the problem.

If, on the other hand, the following statement is given as a sample solution, then the example would have been more constructive:

> A pentagon can be divided into two figures, a triangle and a quadrilateral.

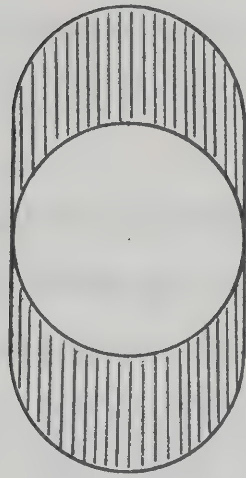


In this particular statement, the operation is "dividing a pentagon with straight line segment", and the range is "the resulting geometrical figures." To be sensitive here could then imply the

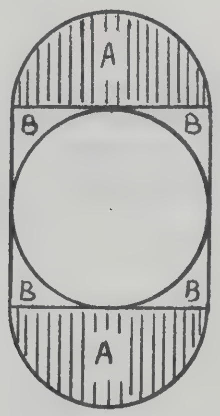
ability to abstract and see alternatives. Students can divide the pentagon with more than one line, with line(s) not joining two vertices, or with curve(s) instead of straight line(s). Others may even recombine the various pieces into new polygons. Conjecturing problems of this type thus also inevitably reflect the processes of sensitivity and redefinition.

In short, it has been argued that creative problem-situations can be meaningfully described in terms of the three parameters D, O, and R. To give, as Boychuk has done, only a single parameter without explicit instructions or implications about the characteristics of the other two parameters is likely to confuse the students. This defeats, in turn, the aim of obtaining reliable information about the level of creativity of the students. Specifically, three types of creative problem-situations have been identified for the purpose of this study. These are discussed below with the help of examples:

(a) D and R are given in precise terms, and O to be found. Two possibilities arise. The situation can be such that besides the obvious routine solution which is the least effective and least efficient approach, it can be redefined in an equivalent form, solvable through a very simple way. Alternatively, it can be a situation that cannot be solved given the students' resources. However, through some form of transformation to an equivalent situation, a simple solution can then be obtained. The best example is Wertheimer's (1959: 266-8) famous "altar-window problem":



Given the relevant dimensions, the students are asked to find the area of the shaded parts. The routine solution is to divide the figure into three parts:

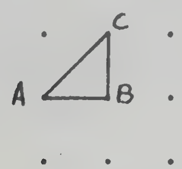


First obtain the total area (A) of the top and bottom semicircles. The area (B) of the sum of the four small pieces of the middle square is obtained by taking away the central circle from the area of the square. The area of the whole shaded area is then the sum of A and B. However, if we notice that the two semicircles (A) just make up the middle circle, then the required area is exactly that of the middle square. In the event students have not learned the formula for the area of a circle, they will be obliged to transform the required area into a square in order to solve the problem. There is no

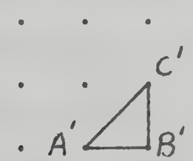
doubt about the need of "sensitivity" to spot the transformation "trick", with "redefinition" as the next process required to complete the calculation.

(b) D (or R) is given in precise terms, with O and R (or D) broadly defined to delimit the "universe of discourse" within which the problem is to be tackled. For example:

(i) Find all the possible images of $\triangle$ABC within the 9-dot region. Vertices of images must lie on given dots. State also the motions that map the original onto the images.



(ii) Find all the possible positions of the original $\triangle$ABC with vertices on the given dots, and whose image is the given $\triangle$ A'B'C'. State also the motions that map the original onto the images.



Generally speaking, these two examples are conjecturing-type problems. However, both sensitivity and redefinition may be involved in the solution process. For instance, we see that within the 9-dot region, there are exactly 16 triangles (which constitute the R) congruent to $\triangle$ABC in (i). If the students are aware of this fact, they can immediately transform the problem into the simpler one of "finding the appropriate O's (motions)." A more sensitive student will notice the operation can be a single motion (a slide,

reflection, or turn) or a combination of motions. Sensitivity leads to a quick discovery of many solutions of the R set as well as the O set. Furthermore, redefinition is implied in the process of discovering the 16 triangles and reversing the routine way of working from the given D through the operation O to reach the R. A similar situation obtains for example (ii).

(c) O is given in precise terms, with D and R broadly defined to specify the "universe of discourse". For example,

> Draw many closed geometrical figures with vertices on the given dots, and find all the possible single-reflections that will map each of the figures onto itself.

To solve the problem, a student only has to notice that this is essentially a problem that is asking for the various lines of symmetry of a geometrical figure. Here, sensitivity enables her/him to see the relation between identity-reflection and the lines of symmetry, whereas redefinition is entailed in reduction of the problem to the equivalent one of locating the lines of symmetry of a figure.

In sum, the above discussion of the three types of creative problem-situations lead to the following conclusions:

(a) In any creative problem-situation, we can never be sure which single process or sequence of processes will be used by a student to solve the problem. Hence, it is invalid to argue directly

from a student's responses to a creative problem which purports to elicit a particular process that she/he is in fact exhibiting that specific process.

(b) The functioning of any one of the three creative processes in different problem-situations are definitely quite different. In other words, there is no obvious common underlying mental capacity that enables one to respond adequately to different examples of the three types of creative problem-situations, whether in terms of sensitivity, redefinition or conjecturing. It is probable that the mental skills involved in the functioning of any one of those creative processes is problem-specific. That is, there are hardly any general creative skills which would enable one to sense deficiencies or possibilities in all creative problem-situations. Neither is there a class of general mental skills that would help one to simplify all problems that need redefinition or transformation. Consequently, instructional programs designed to enhance students' creative abilities in sensitivity, redefinition and conjecturing are unwarranted and unlikely to have any effects on students' mathematical creativity.

(c) In lieu of Boychuk's (1974: 59, 73-74, 85, 93) special guidelines for the construction of exclusive types of problem-situations reflecting each one of the creative processes (which we deem inadequate and impractical), we have successfully demonstrated that there are three types of creative problem-situations defined in terms of the three parameters, D (domain), O (operation) and R (range). Such problems will elicit the processes of sensitivity,

redefinition and conjecturing. The task therefore is to construct creative instructional materials and test items following the guidelines defined by these three creative types.

OPERATIONAL DEFINITION AND SCORING CRITERIA

The Discrepancy Issue

When Dunn (1975: 331) asserts that "the basic problems relating to acceptable definitions of creativity in terms of recognizably relevant criteria are still to be solved," he is essentially referring to the "discrepancy" between various "conceptual" and "operational" definitions of creativity (Crockenberg, 1972: 40). While "a unified, widely-accepted theory of creativity (Treffinger, Renzulli and Feldhusen, 1971: 107)" is still non-existent, there is nevertheless a general consensus with regard to the "operational definition" of creativity. The various conceptual definitions of creativity basically postulate the existence of intellectual processes, personality traits or other psychological constructs in human behaviors. These psychological constructs, however, are not directly observable, and their existence can only be inferred by psychologists from intellectual production under certain conditions. In order to identify the creative worth of these intellectual products, which serve as indicators of their postulated psychological constructs, "operational definitions" or "creative criteria" are needed.

Definition and Criteria

Most creativity researchers will probably agree that there are at least three kinds of response (intellectual product) properties which can serve to judge creative behavior. Boychuk (1974) lists them as fluency, flexibility and originality, or fluency, variety and novelty. Others (Jackson and Messick, 1965; Treffinger, Renzulli and Feldhusen, 1971; Crockenberg, 1972) argue that single measures of fluency, flexibility and originality are not sufficient, and additional criteria are required, such as transformation, elaboration and condensation.

> While it may be eminently reasonable to study the processes that appear to be involved in creative production, it is conceptually unjustifiable to call these tests "tests of creativity". To be entirely fair, most psychologists are aware that fluency is not identical to creativity and that there are additional stages in the creative process beyond the initial generation of ideas (Crockenberg, 1972: 40).

Without doubt, much of the controversy concerning the congruence between creative processes and creative productions can be related to the following difficulties: problems in the conceptual and operational definitions of creative behaviors, the validity of the test items as problem-situations reflecting postulated creative processes, and the validity and reliability of the scoring procedures utilized. In order to resolve these problems, we have in the previous sections adopted a Three-Process Creative Problem Solving Model as our conceptual definition of mathematical creativity (supra: 46f). Using illustrations from school mathematics, we next showed how creative problem-situations

can be constructed to reflect the functioning of the three processes of mathematical creativity defined in the chosen conceptual model (supra:60f). When students respond to these problem-situations, their divergent productions can then be reasonably regarded as the manifestations of their mathematical creativity defined in terms of sensitivity, redefinition and conjecturing. However, as previously indicated, these processes cannot be identified separately from the student's creative responses. There is hence a need to establish criteria for evaluating the responses such that the latter can be justifiably argued as validly reflecting students' mathematical creativity. Such criteria constitute the operational definition of creativity used in this study.

It has been demonstrated that the key elements of creative problem-situations are (1) the flexibility of the implications of information given, and (2) the requisiting of a variety of responses to a given problem. Creative students are thus those who will produce many different responses (fluency), many different "kinds" of responses (flexibility or diversity), and many uncommon "kinds" of responses (originality or rarity). In sum, for this investigator, creativity is operationally defined as consisting of the factors of Fluency, Diversity and Rarity of ideas.

A three-fold scoring scheme for the creative production (responses) is therefore required. Briefly, it comprises (1) a fluency score which is the total number of acceptable answers with duplicates eliminated, (2) a diversity score which is the total number of relevant and distinct categories of mathematical concepts

and/or creative behaviors involved, and (3) a rarity score which is the total weighted scores according to statistical infrequency of the "diversity categories" within the overall categories produced by the sample. Elaboration of each dimension of this scoring scheme will be given in Chapter VI.

DOMAIN OF KNOWLEDGE

Intuitive and Informal Geometry

The present study represents an attempt to enhance the mathematical creativity of junior high students through special creative instructional strategies. It is therefore important to spell out clearly the domain of knowledge and its role in the conceptual framework.

By the mid-sixties, there was evident profound changes in both the content and structure of school mathematics, as well as in the methods of instruction. Indeed "these new methods (discovery, emphasis on proof, etc.) do not stand above but are to be regarded as essential concomitants of the new contents" (NCTM, 1964: 78-79). On the other hand, a new approach may also prescribe to some extent the form and content of instruction. Furthermore, as some mathematics educators repeatedly emphasize, any research concerning method or content should take into consideration the classroom situation.

> The role of any item of content or of procedure depends upon two things, its potential value with reference to the goals of mathematical instruction, and the effectiveness with which the item is

> incorporated into the classroom teaching process (Jones, 1966: 106).
>
> ... education begins, and ends, in the classroom and that any researchers who loses touch with that 'real' situation is in danger of losing touch with education (Bishop, 1972: 17).

In effect, the "domain of knowledge" selected for this study should ideally allow an optimal interaction of content, method and classroom situation during the creative instructional process. Geometry would seem to be a reasonable choice for this purpose. Junior high school geometry, in particular, emphasizes the spatial intuition through informal and plausible reasoning, hence providing optimal opportunity for creative thinking. Coxeter (1967: 9) claims that,

> In geometry, perhaps more than in the other subjects, a student can exercise originality and ingenuity in devising a construction or seeking a proof.

Since 1890, geometry as a branch of the school mathematics curriculum in the United States has been receiving heavy criticism and re-evaluation (Quast, 1968). Because of the dissatisfaction of many mathematics educators with traditional Euclidean approaches, new school programs on geometry have continually come into being. Most critics argue that there is neither mathematical nor pedagogical reason to substantiate the standard, formal and axiomatic approach to school geometry programs (Trafton and LeBlanc, 1973: 43). Instead, they recommend that "the intuitive 'interest' approach through problems significant to the student" (Coxeter, 1967: 9) be adopted.

Such interest in "informal" and "intuitive" approaches to geometry thus led to the introduction of other types of geometries into school mathematics. One of these is based on the ideas of transformational geometry or motion geometry.

Why Motion Geometry

The study of "transformations" in geometry was first advocated by Felix Klein in his Erlanger Program as early as 1872 (Fehr, 1973: 371). Transformational geometry provides a dynamic comprehension of the spatial relations in physical world, in contrast to the static geometry of Euclid. This accounts for its name of "motion" geometry in school curriculum. In the United States, some mathematics educators (Schuster, 1967; Adler, 1968; Allendoerfer, 1969) have suggested that the study of geometric transformation be one of the major topics of tenth-grade geometry. In England, the School Mathematics Project emphasized "motion geometry" for study in Grades 6-8 (Thwaites, 1966; Elliott, 1967). Of late, therefore, there has been a gradual awareness of the value of "motion geometry" as subject content for school mathematics learning.

We will now argue that "motion geometry" constitutes a highly suitable subject content for our study. Four basic pedagogical questions are considered crucial to the objectives of this research in creative instruction:

(1) Can content topics in motion geometry provide rich problem-situations for creative instruction in junior high mathematics?

(2) Are these topics <u>teachable</u> at junior high level?

(3) What <u>effect</u> will these contents have <u>on other</u> work in <u>school mathematics</u>?

(4) Is motion geometry <u>related to</u> the <u>classroom reality</u>?

A consideration of these questions will provide the rationale for the utilization of motion geometry as our instructional content materials.

(1) <u>Content and Creative Instruction</u>

The content of traditional school geometry based on a static, formal and rigorous Euclidean approach, is incompatible with our conceptual and operational definitions of mathematical creativity which emphasize inventive and divergent production of ideas. On the other hand, the dynamic nature of motion geometry lends itself admirably to an informal and intuitive approach which stresses sensitivity, conjecturing, transformation, and inquisitivity (Peterson, 1973: 60). Beginning with very simple notions of slide, reflection and turn, motion geometry can lead to the exploration of abstract mathematical concepts of congruence, symmetry, similarity and parallelism, and hence further enrich the "students' geometrical experience, imagination and thought" (Fletcher, 1969: 270). This exploration can be carried out in an intuitive and flexible manner that provides innumerable opportunities for students to make test conjectures (Sanders and Dennis, 1968: 369). In effect, the versatility, variety and range of problems and solutions appropriate to motion geometry make the subject matter a rich area for creative thinking (Del Grande, 1972).

(2) Teachability

Instructional strategies and materials can be and have been developed centering on the concepts of transformations (motions) on geometric figures (UICSM, 1969; Coxford and Usiskin, 1971). These and other similar materials have been found to be adequate in school mathematics teaching. Usiskin (1970) concludes from his experiment that grade ten students can attain competence with geometric concepts unique to the transformational approach at a level of comprehension equal to that attainable with traditional Euclidean formal approach. Sigurdson (1974) has demonstrated the feasibility of successfully teaching motion geometry to grade nine students in an informal, exploratory and inventive approach ("mathematizing mode"). Such studies therefore indicate that motion geometry is an appropriate topic for students, since movements of figures and shapes in the plane and space can be easily studied (Trafton and LeBlanc, 1973: 43).

(3) Relation with Other Mathematics

The issues of prior mathematics and subsequent mathematics are involved in the introduction of motion geometry to school mathematics. Since basic concepts of transformation and congruence can be taught through students' intuitive notions (e.g. "movement", "same size" and "same shape"), no apparent prerequisite is necessary in order to implement the transformational approach to geometry. This argument has been substantiated by the studies cited above and other experiments (Servais and Varga, 1971: 53-60; Coxford, 1973: 195-198).

As far as subsequent mathematics is concerned, concepts of geometric transformation furnish a good foundation for further development of other mathematical concepts. This is because these concepts (a) serve as a natural introduction to concepts of mapping and function, (b) furnish a concrete basis for early study of vectors, (c) give a simple formulation of the idea of congruence and (d) provide an excellent example of the mathematizing of the physical world, through the notion of isometric transformation as a mathematical abstraction and generalization of physical spatial relations (Schuster, 1973: 391).

(4) Relation with Classroom Reality

Almost all research in teaching aim at effecting changes in classroom instruction. Results of instructional studies which incorporate many elements of classroom reality contribute most to the evaluation and improvement of students' actual learning situations. A strong argument has thus been made by Getzels and Dillon (1973: 718) that instructional strategies based on contents and procedures not part of the reality of school situations are often not directly applicable to classroom practice. The recent introduction of motion geometry to the Edmonton Separate School Mathematics Program therefore provides this study with a realistic situation for conducting research.

In short, on the basis of these four major considerations, motion geometry was selected as a highly appropriate and promising subject content for this investigation.

CREATIVE INSTRUCTION

Encourage Divergent Thinking

The key element in the creative problem solving model of this study is that of divergent thinking: the generation of a great variety of ideas related to the solutions of a given mathematical problem-situation. In the literature review on methods of creative training, we have identified the same fundamental element in most of the creative approaches and training programs. Consequently, this study is based on the fundamental assumption that creative problem solving can be learned by junior high students through a creative instructional method which encourages "divergent production". Furthermore, it is obvious that mathematical creativity can only be developed in "creative" mathematical problem-situations. Using these same assumptions, Gray and Youngs (1975: 291) have developed creative teaching strategies utilizing Guilford's "divergent production matrix". They assert that

> If teachers can learn how to facilitate the generation of multiple hypothesized unique solutions on the part of their pupils, they will have overcome one of the most frustrating obstacles to creative learning. Of course, ability to facilitate hypothesizing is not an end in itself -- and must be learned in a context which clearly reveals its relationship to the total problem solving process.

Besides, the encouraging of a great variety of ideas in concrete mathematical situations, there are other factors to be considered. Reviewing the results of 142 studies designed to assess the effects of various creative approaches, Torrance (1972: 132-133) concluded that,

> The most successful approaches seem to be those that involve both cognitive and emotional functioning, provide adequate structure and motivation, and give opportunities for involvement, practice, and interaction with teachers and other children. Motivating and facilitating conditions certainly make a difference in creative functioning but differences seem to be greatest and most predictable when deliberate teaching is involved.

It can be assumed, of course, that cognitive skills, appropriately structured problem-situations, motivation, interaction and environmental conditions conducive to divergent production, all play an important part in the development of students' mathematical creativity. However, at the present stage of research in mathematical creativity, any attempt to incorporate all of these variables would be unwarranted and practically unmanageable, owing to the lack of commonly agreed operational definitions about most of these variables. On the other hand, we have demonstrated the usefulness of conceptualizing instruction in terms of the two basic functions of <u>display</u> and <u>control</u> (<u>supra</u>: 27ff). Hence we now proceed to discuss the proposed instruction along these two dimensions. An attempt will be made to incorporate some of the motivational and environmental variables emphasized by Torrance (1972: 132-3).

Two Functions of Instruction

(1) <u>Display Function</u>

The creative solution of mathematical problems requires utilization of relevant mathematical concepts, rules, principles and operations. Parnes (1963a, 1963b, 1963c) found that the major obstacles to creative production seem to be the absence or

inaccessibility of prerequisite knowledge. Creativity seems to be facilitated when prerequisite knowledge and skills have been provided or recalled. Gagne and Briggs (1974: 12, 172) assert that "effective learning and problem solving require the accessibility of information" and all students should have the same opportunity to be creative, and that "their solutions are not handicapped by the absence of necessary knowledge and intellectual skills." Our analyses of studies on "mathematizing mode" in Chapter II also indicates the importance of prerequisite mathematical knowledge in solving creative divergent types of problems. Hence it is necessary to introduce the basic concepts and processes of motion geometry via a direct and effective method before any form of creative instruction.

(2) Control Function

Students' classroom behavior is directly or indirectly regulated by the teacher's behavior. Gallagher and Aschner (1963) found that slight teacher encouragement is enough to evoke an array of divergent students' responses. Emphasizing the need to reward students' creative ideas, Torrance (1966c: 165-166) claims that,

> Educational research has been quite strong and consistent concerning the fact that people tend to achieve along whatever lines they find rewarding. If achievement tests require creative responses, textbook writers and teachers will be motivated to find ways of encouraging this kind of development, and students will store information in such a way that it can be used in producing creative applications, making judgements, and decisions, and the like.

Clearly, an excellent place where a teacher can provoke and

reward creativity is assigning creative problems as exercises. Since the construction of problem sets is designed to illustrate text materials and provide practice for students, it is reasonable to argue that the most carefully constructed sequences of traditional type problems would not be appropriate for the new creative approach and objectives (Dilworth, 1966: 91). Mathematical creativity as defined in this study requires students to generate many solutions to any given problem. Hence there is a need to construct special sets of creative exercises with the following four objectives:

(1) To provide the kind of problem-situations which would call upon the three creative processes: sensitivity, redefinition and conjecturing.

(2) To provide creative problem-situations which explicitly ask for more than one solution, thus making clear that divergent and creative responses are rewarded.

(3) To establish an environment where divergent responses become part of the normal classroom practice, hence providing motivation for creative behavior.

(4) To provide opportunity for practice in solving mathematical problems creatively, that is in more than one way.

Our emphasis on the role of creative exercises is supported by the Cambridge Conference on School Mathematics (1963: 19), which claims that,

> ... they (exercises) are the most important part of the prepared course material. The exercises should guide the student, and also the teacher, to the meaning and relevance of the theorems and concepts; they should train and develop the skills, stimulate creative thinking, and develop ingenuity.

Consequently, the creative instruction for our study will be developed along the two major dimensions of classroom instruction: (1) the display of essential mathematical concepts and skills as basic knowledge for creative mathematical activities, and (2) the control of students' mathematical activities through creative and divergent types of exercises. Details of the instructional method and materials will be given in Chapter IV.

SUMMARY

In this chapter, the conceptual definition of mathematical creativity for our study was defined in terms of three processes: sensitivity, redefinition and conjecturing. This is a modified version of Boychuk's (1974) theoretical model of creative problem solving in school mathematics. We have demonstrated the existence of a logical discrepancy between the conceptual and operational definitions of most of the studies in creativity. To resolve this discrepancy issue, the present study proposed to show the logical connection between our conceptual definition of creativity and the mathematical problem-situations designed to reflect the three creative processes.

The view that problem solving is a form of rule-governed behavior was shown to provide better denominators for characterizing problem-situations reflecting the three creative processes. Creative problem-situations can thus be described meaningfully in terms of sets of domain (D), operation (O) and range (R). Such a

description was claimed to provide (1) more practical and useful guidelines for the construction of creative problem-situations, and (2) a logical linkage between the conceptual definition in terms of the three processes and the operational definition in terms of the three factors: fluency, diversity and rarity of ideas.

It was also argued that motion geometry with its intuitive, informal and dynamic features lends itself admirably to creative instruction. Finally, we proposed that the creative instruction used in the study should emphasize the two major functions of classroom instruction: (1) to display basic concepts and skills as prerequisite, and (2) to control students' mathematical activities through creative exercises.

CHAPTER IV

DEVELOPMENT OF INSTRUCTIONAL METHODS AND MATERIALS

RESULTS OF THE PILOT STUDY

In the summer of 1975, an exploratory study was conducted (a) to aid in the development of the instructional materials and evaluation instruments, (b) to assess the appropriateness of these materials and tests, and (c) to choose the most promising and feasible instructional approach for grade eight students.

Three classes of 61 grade eight students of St. Clare Separate School in Edmonton were taught a unit of Inventive Motion Geometry by a mathematics teacher of the school and this investigator. The pilot study consisted of 16 teaching-learning sessions, each lasting between 40 and 45 minutes, plus two testing sessions at the end. For control purposes, another 39 grade eight students from two classes of St. Pius X Separate School in Edmonton, who had not been exposed to "Motion Geometry" or "Inventive Motion Geometry" were given the same creativity test. Group differences between the students of the two schools could therefore be compared.

Content Domain

Concepts and skills of motion geometry presented during the pilot study were as follows:

(1) Slide:

(a) Notions of the original, slide arrow, slide image, and standard notation: (___R/L, ___U/D).

(b) An original and its slide image are congruent.

(c) Line segment and its slide image are parallel.

(d) Line segments joining points and their images are parallel and congruent.

(2) Reflection:

(a) Notions of the original, mirror line, or line of reflection, and reflection image.

(b) An original and its reflection image are congruent.

(c) A point and its reflection image are equi-distant from the mirror line.

(d) The mirror line is perpendicular to and bisects the line joining a point and its reflection image.

Inventive Method

The instructional approach used was the Inventive Method (IM) which will be discussed in detail in the next section. Basically, the IM consisted of two phases: (1) the Development of Concepts (DC), and (2) the Inventive Discussion (ID). During the DC phase, the teacher introduced the needed basic concepts of a slide or a reflection to the class by means of an approach similar to

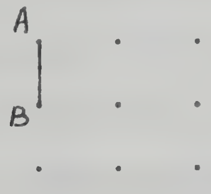
that suggested in the Program (Edmonton Separate School Board, 1975) (infra:98ff). Ample classroom practice was then given to the students to ensure mastery of these geometric concepts and skills. This was followed by the ID of two types of Inventive Exercises (IE): (1) Divergent Type, and (2) Process Type.

Inventive Exercises (IE)

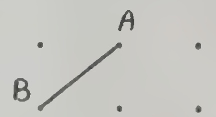
(1) Divergent Type of Inventive Exercises. This type of "open-problem" emphasizes the production of many solutions to any given problem-situation. Some or all of the three creative processes, sensitivity, redefinition and conjecturing might be used, but these processes were not discussed explicitly during the ID phase. The following are a few examples of this type of inventive exercises used.

(a) If image segments have end-points on given dots, draw all the slide images of $\overline{AB}$ in each of the following:

(i)



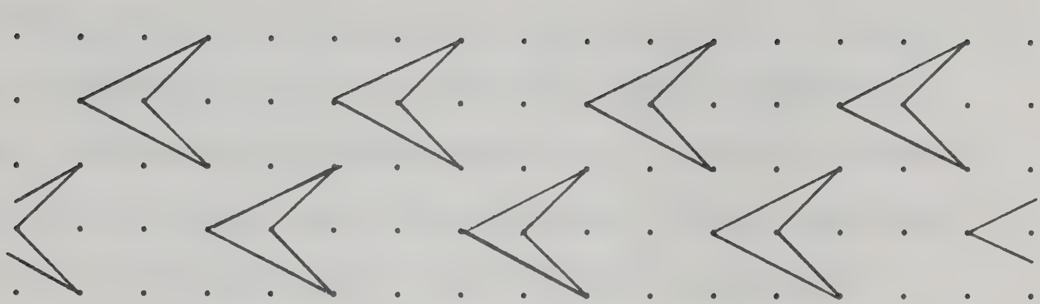
(ii)



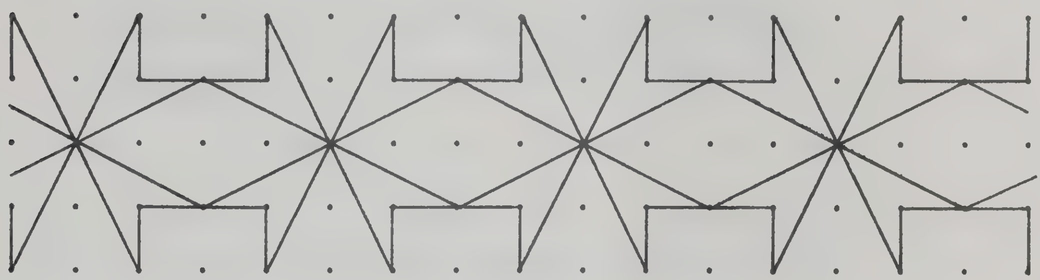
(b) $\triangle A'B'C'$ is the slide image of $\triangle ABC$. Join each point with its image point with dotted lines. Put down all the important mathematical properties you have noticed. Give as many statements as possible. Here is one example:

$$\overline{AB} \cong \overline{A'B'}.$$

(c) What slide leaves the strip pattern unchanged? Give as many different slides as you can think of:



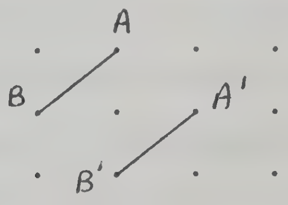
(d) What are the possible Basic Patterns that generate this strip? Give all the possible basic patterns you observed, and show how you generate this strip through appropriate motions applied to your basic patterns:



(2) Process Type of Inventive Exercises. This type of creative problem requires explicit use of some of the three creative processes to arrive at an acceptable solution or solutions. Some examples are given below together with the sort of brief instructions used to guide classroom discussion (ID).

(a) Sensitivity-Problem:

Here is a given region with 12 dots. $\overline{A'B'}$ is the slide image of $\overline{AB}$. How many different ways can you slide $\overline{AB}$ onto $\overline{A'B'}$?

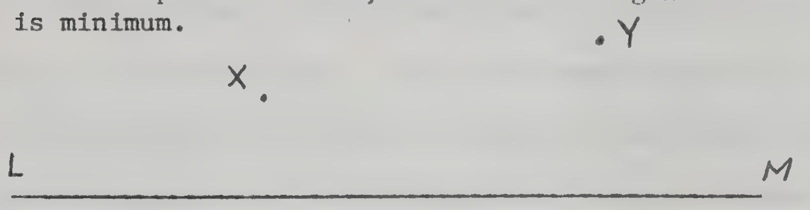


All the students gave only one obvious solution to this sensitivity-problem, i.e. the slide (1R, 1D). Through discussion,

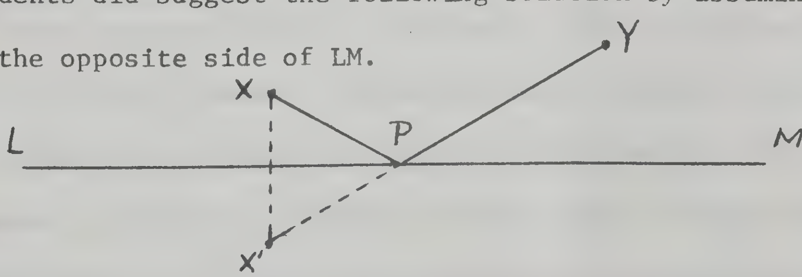
the teacher attempted to bring the students' attention to the fact that no restriction had been placed on the number of permissible slides. Consequently, successive slides were produced by students to explore the various possible combinations. Students were also made aware of the importance of "sensitivity" by looking for "possibility", "shortcomings", "deficiencies", and "alternative interpretations".

(b) Redefinition-Problem:

X and Y denote the position of two farmhouses.
LM represents a main cable supplying electricity.
Locate a point P on LM, so that the length XPY is minimum.



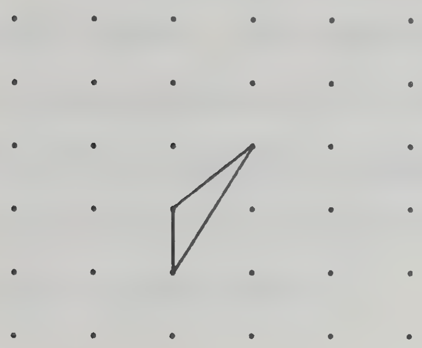
Students were prompted to redefine the problem in some convenient form which could be solved easily such as changing the position of the line LM, or the position of X or Y. Eventually, some students did suggest the following solution by assuming that X is on the opposite side of LM.



If X is on the opposite side of LM, i.e. at X', then X'PY, being a straight line is the shortest distance from X' to Y. By reflection property, X' is the image of X in LM, and X'P is the reflection image of XP, therefore, XP + PY = X'PY, is shortest.

(c) Conjecturing-Problem:

You are given a basic pattern within those given dots. Form as many different strip patterns as you can within those dots:



In this problem, students could use a slide, a reflection or a combination of both motions. When they were prompted to look carefully at the domain of freedom defined, a few of them even suggested the notion of a "Turn" (rotation), though without precise mathematical terms.

Implementation

Both the participating teacher and this investigator were present in all the lessons, though the former taught two classes and the latter one. There were short orientation sessions before each lesson, and brief discussion sessions after every lesson. The teacher was very helpful and enthusiastic, and many constructive suggestions and new ideas were presented during those sessions.

At the end of the 16 instructional sessions, a creative geometry test consisting of 6 of Boychuk's (1974) creative problems with modifications was administered to all the 61 students in two sessions of 45 minutes each. For the purposes of comparison, a

control group of two classes with a total of 39 grade eight students in St. Pius X Separate School was given the same creative test. No creative "motion geometry" test was constructed for the pilot study for two reasons: (1) examination of students' written responses to assigned sets of IE indicated that students could react meaningfully and creatively to divergent type problem-situations; and (2) owing to its content-specific nature, such a test could not be given to the control group.

Results and Implications

This exploratory study yielded a number of observations and conclusions which allowed appropriate modifications to be incorporated into our previous discussion on the conceptual framework (supra:40ff) and the evaluation design for our proposed instructional study.

(1) Though the participating teacher was enthusiastic about the Inventive Method (IM) of teaching motion geometry, he felt the need to formalize the IM procedures so as to guide more precisely the teacher's classroom behavior. He preferred the "Divergent Type of Inventive Exercises" over the "Process Type", which depended considerably upon student responses and required the teacher to react extemporaneously to a great variety of unexpected responses from students, while simultaneously leading them towards the convergent solution through the specific creative process. Thus the Inventive Exercises (IE) in the main study have been designed to contain merely Divergent Type problems which emphasize

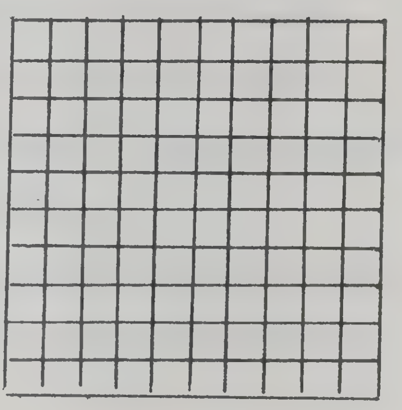
the generation of a great variety of solutions.

(2) The results of the "creative geometry test" indicated the effects of the <u>Divergent Type</u> of IE. Three of the test items were typical problems of this type:

(a) <u>Sensitivity-Problem I</u>:

Cut the square in half.
What are the shapes of the resulting halves?

Draw a diagram to show your reasoning.

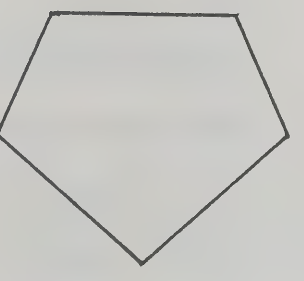


(b) <u>Conjecturing-Problem I</u>:

You are given a Pentagon.
Make as many conjectures as you can about a pentagon.
For example:

"A series of pentagons cannot cover a flat surface without gaps unless the pentagons overlap."

Now make some of your own conjectures.

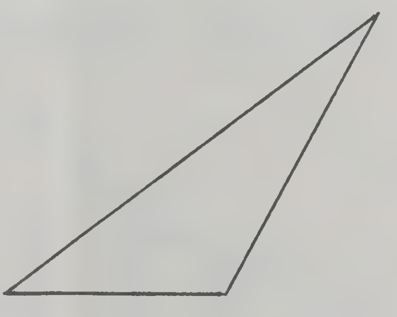


(c) <u>Conjecturing-Problem II</u>:

You are given a Triangle.
Make as many conjectures as you can about a triangle.
For example:

"Two congruent triangles make a parallelogram."

Now make some of your own conjectures.



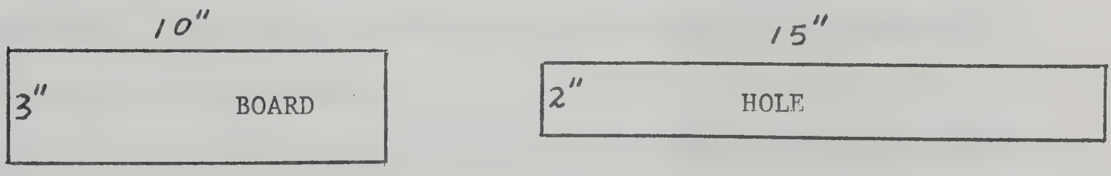
Though the mathematical background of students of the two schools did not seem to show any obvious superiority on the part of either school, students of St. Clare Separate School produced twice as many responses for these three problems than students of St. Pius X Separate School. This result thus establishes the effectiveness of the IM of instruction in enhancing students' creative ability to generate a great variety of ideas as measured by the criteria of fluency, diversity and rarity.

(3) As far as the other three process type of convergent problems were concerned, students of both schools did poorly. These problems are as follows:

(a) Sensitivity-Problem II:

Cut the Board into two equal pieces that will cover the Hole completely.

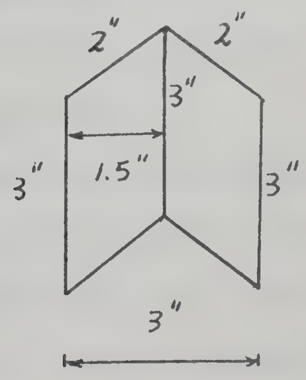
Show all your attempts including your incorrect ones.



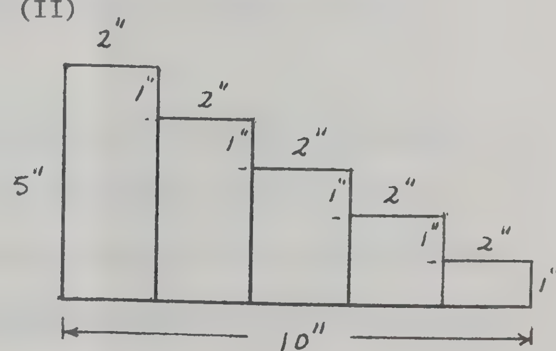
(b) Redefinition-Problems:

Find the areas of the following figures. Show all your steps.

(I)



(II)



In the sensitivity-problem II, out of 100 students from the two schools, only one student successfully solved the problem. For the two redefinition-problems, students either calculated the areas wrongly or used the standard routine methods to obtain the solutions. A plausible explanation of this performance is that such convergent type of creative problems, which demand transformations in definite directions in order to resolve the situations effectively and efficiently, have "problem-specific" properties varying from problem to problem. Training in one type of sensitivity or redefinition problem-situation did not therefore promote transfer to another type of problem. Nevertheless, it is noted that most of the students who were exposed to the IM of instruction seemed to make more attempts in solving the Sensitivity-Problem II, while some gave two or three alternative solutions (though routine type) to the two redefinition-problems. This might be taken as an indication of the positive effect of the IM treatment.

(4) The frequent requests from students during the testing sessions for clarification of the requirements of the two conjecturing-problems indicated the ambiguity of these "open" problem-situations. This problem has been overcome by constructing problem-situations in terms of the three parameters: domain (D), operation (O), and range (R) (<u>supra</u>:56ff).

In sum, the "inventive" instructional procedures were shown to be moderately successful in terms of enhancing students' ability to generate many ideas to a given problem-situation. The pilot study showed that the specially designed Inventive Method (IM) was feasible

and promising, and provided direction for incorporating required improvements into the design of the two different instructional methods and instructional units in the following sections, as well as for the construction of test instruments to be discussed in Chapter VI.

TRADITIONAL AND INVENTIVE METHODS OF INSTRUCTION

On the basis of the conceptual framework discussed in the previous chapter, as well as the information obtained in the pilot study, it was deemed feasible and promising to structure instructional methods along the two dimensions of display and control. In addition, since our purpose was to assess the effectiveness of a proposed creative instructional approach by comparing it with that of a conventional approach, two distinct instructional methods were developed: (1) the Traditional Method (TM), and (2) the Inventive Method (IM). Corresponding to the two instructional dimensions, both TM and IM consisted of two phases: (a) the Development of Concepts, and (b) the Discussion of Exercises.

Display: Development of Concepts

It has already been pointed out that to be creative in mathematical problem-situations requires the accessibility of necessary prerequisite mathematical concepts and skills. Moreover, in order to be able to assume with confidence that all students in

this study have in fact mastered the relevant mathematical knowledge (Gagne and Briggs, 1974: 172), both the Traditional Method (TM) and the Inventive Method (IM) would have to have the same Development of Concepts (DC) phase of instruction. In other words, both TM and IM classes would learn the same mathematics content in the same way. In this regard, we have followed DeCecco's (1968: 468) advice that expository teaching can effectively and efficiently present a rich body of content materials. This view is substantiated by some well-designed experimental studies in naturalistic classroom settings (Tobert, 1969; Worthen, 1968), where expository classes out-performed discovery classes on achievement tests which assess students' initial learning of concept knowledge. Consequently, the Development of Concepts phase of both IM and TM would be structured on the traditional, direct expository method.

Basically, such a design attempts to expose students of the two treatment classes to the same amount of "content" instruction under similar teaching method and learning environment, and hence equal opportunity to learn the same basic mathematical concepts and skills. In effect, having the very same DC phase of instruction for the two treatments provides adequate "experimental control" to the study. This strengthens the "internal validity" of the study by eliminating other plausible rival hypotheses which would attribute any differences in students' ultimate achievement to factors other than the effects of the treatment (or instruction).

Control: Discussion of Exercises

Exercises

As indicated earlier (supra: 28), teacher's classroom behavior directly or indirectly regulates and controls students' learning behavior; and, assigning and discussing specially constructed sets of exercises are two of the best ways to encourage and reward creative thinking in students. It was argued that new sets of the divergent type of problems were needed to stimulate creativity. Two sets of exercises were therefore constructed for the two different instructional methods: (1) Traditional Exercises (TE), and (2) Inventive Exercises (IE).

(1) Traditional Exercises (TE): sets of conventional convergent type of problems which ask for single correct solution to each problem-situation.

(2) Inventive Exercises (IE): sets of convergent and divergent problems constructed for the Inventive Method:

(a) Convergent-type of problem-situations are similar to that of TE of the single-answer type whose purpose is to ensure that students have enough practice on the basic concepts and skills learned in the DC phase of the instruction.

(b) Divergent-type of problem-situations are creative problems constructed under the guidelines established earlier (supra:56ff) to fulfil the four objectives of providing situations reflecting the three creative processes, situations demanding divergent productions, a creative learning environment, and practice in creative situations (supra: 77).

It should be pointed out that "inventive" is used instead of "creative" to qualify both the instructional method and the constructed sets of exercises, in order to emphasize the distinction between the basic conceptual definition of creativity and the particular divergent nature of the exercises. These Inventive Exercises are not designed to train specific intellectual skills in sensitivity, redefinition and conjecturing, but rather to enhance fluency, diversity and rarity of ideas in students.

Details of the construction of these sets of Traditional Exercises and Inventive Exercises are given in the latter section of this chapter.

Discussion

This component is stipulated to stress the importance of these sets of exercises, and to ensure their full utilization. Essentially, it requires a class discussion of previously assigned exercises (TE or IE) before the "Development of Concepts" phase of each new lesson. Thus an opportunity is provided for stimulating class interaction among students, and with the teacher. Such discussion also would further reinforce the creative environment in the creative learning situation.

In summary, the two distinct instructional approaches are:

(1) Inventive Method (IM): This is a sequence of instructions consisting of (a) the Development of Concepts (DC) and (b) the assigning of a set of Inventive Exercises (IE) as homework, and (c) the Inventive Discussion (ID), which is the

classroom discussion on the assigned IE. The IE were specially designed to contain both inventive divergent and conventional convergent types of problems. The divergent-type problems encourage the generation of many appropriate solutions to a given situation.

(2) Traditional Method (TM): This is a sequence of instructions consisting of (a) the Development of Concepts (DC), (b) the assigning of Traditional Exercises (TE), and (c) the Traditional Discussion (TD), which is the classroom discussion on the assigned TE. The TE given in the *Program* (Edmonton Separate School Board, 1975) contain mainly convergent single-correct-solution type of problems.

TRADITIONAL AND INVENTIVE UNITS

The development of two distinct instructional methods necessitates the need to develop two separate instructional units. Hence two units on grade eight motion geometry were constructed: (1) the Traditional Unit, and (2) the Inventive Unit. Both units consisted of 19 lessons, each of which was made up of two parts: the Development of Concepts and the Exercises. Two different sets of exercises were designed for the two units: the Traditional Exercises (TE), and the Inventive Exercises (IE).

Content Domain

Earlier, we argued that junior high motion geometry constitutes one of the most appropriate subject contents for inventive instruction. The dynamic nature of geometric transformations with such notions as congruence, symmetry and similarity provides rich problem-situations that reflect the creative processes of sensitivity, redefinition and conjecturing. Moreover, the novel introduction of "motion geometry" in Edmonton Separate School System means that we have a good experimental environment where the situation has not been contaminated by previous learning about the same subject content.

Clearly, the creative thinking in mathematics as defined in this study is contrary to the current way of thinking in school mathematics in the Edmonton Separate School System, where "convergent single-answer" frame of thinking is predominant. There is therefore a clear need for a long enough period of instruction and training which will allow students sufficient familiarity with the "creative mode" of thinking. Taking into consideration the schedule of junior high school mathematics teaching in the Edmonton Separate School System, a six-week instruction seemed an optimal period for the purposes of our study. By consulting the participating teacher, we discerned that a unit of 19 lessons could be covered more or less in 6 weeks. Consequently, the instructional materials of the first 19 lessons in the Program were selected as the content domain for our study.[1] These instructional materials were defined in terms

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[1] Lessons 2 to 20 were selected. The first lesson was a revision, not suitable for our purposes.

of the following 19 objectives:

(1) State the properties of a slide: (a) an original and image are congruent with respect to the whole figure as well as the various parts; (b) line segment and image are parallel; (c) line segments joining points and their respective images are congruent and parallel.

(2) State the properties of a turn: (a) an original and its half-turn image are congruent with respect to the whole figure as well as the various parts; (b) a line segment and its half-turn image are congruent and parallel.

(3) State the properties of a reflection: (a) an original and its reflection image are congruent with respect to the whole figure as well as the various parts; (b) a point and its reflection image are the same distance from the mirror line; (c) the mirror line is perpendicular to the line segment joining a point and its reflection image.

(4) State that: (a) a slide-reflection is a combination of a slide and a reflection; (b) that an original and its image are congruent with respect to the whole as well as the various parts.

(5) Use the correspondence notation $[A,B,C,\ldots] \longrightarrow [A',B',C',\ldots]$ to identify a figure and its image under a congruence transformation.

(6) Define the terms regular and diagonals as applied to polygons.

(7) Classify quadrilaterals with respect to their diagonals.

(8) Define acute, obtuse and right angles.

(9) Classify triangles by their angles.

(10) Classify triangles by their sides.

(11) Determine the properties of triangles using the lines of symmetry.

(12) Show that the basic requirements for a pair of triangles to be congruent is one of the following: SSS, SAS, ASA.

(13) Demonstrate that AAA is not a sufficient relationship to produce congruence.

(14) Determine the angle sum of triangle. (Glide the angles to form a line).

(15) Determine the angle sum of polygons.

(16) Identify pairs of angles. (Limit: Supplementary, complementary, linear, opposite, and adjacent.)

(17) Use a motion to prove two lines parallel.

(18) Determine the properties of the angles formed by two parallel lines and a transversal, e.g. use a slide or a half-turn.

(19) Develop a formula for the perimeter of any polygon, e.g. P = sum of sides.

Development of Concepts

Our previous discussion has argued for the need to introduce and develop the basic mathematical concepts and skills via a direct and effective expository approach. The instructional materials and approach suggested in each of the 19 lessons of the *Program* with its highly structured and directed procedures appear to be very useful for achieving this purpose.

According to the *Program*, every lesson consists of two parts: (1) *Development*, and (2) *Exercises*. The "Development" includes some or all of the following sections: (a) *Objectives*, (b) *Possible Materials*, (c) *Prerequisite Skills*, (d) *Suggested Development*, and (e) *Further Suggestions*. The last section was given in some lessons to cater for high ability students. Since the "target population" of this study is all the junior high students of the Edmonton

Separate School System, the special section for highly gifted students may be deemed unnecessary and irrelevant for our purposes. Hence, for both the Traditional as well as the Inventive Units, some or all of the first four sections outlined in the Program would be followed closely to avoid any discrepancy between the two Development (DC) phases of the two instructional methods, TM and IM.

The following is a sample of the Development of Concepts part for one of the lessons, Lesson No. 5. The complete set of 19 DC's can be found in the Program (Edmonton Separate School Board, 1975).

Development of Concepts for Lesson No. 5

Objective: Describe a slide-reflection as a combination of a slide and a reflection, and state that an original and its image are congruent as well as the various parts for the slide-reflection.

Possible Materials: Dot paper, geoboard, overhead, acetate, tracing paper, mirrors.

Prerequisite Skills: Students must know and understand the motions covered in previous objectives, i.e. slide, reflection, and turn.

Suggested Development:

(i) Present diagram 1 to your students; ask them to determine which motion produces the image.

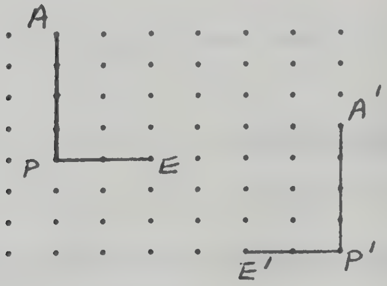


Diagram 1

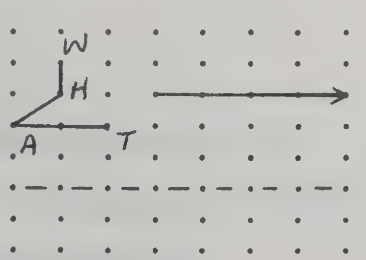


Diagram 2

Students should determine that no single motion could have produced the image.

(ii) Give the students Diagram 2 with the instructions to copy the diagram onto dot paper and then
(a) slide and flip object to produce an image,
(b) returning to the original object flip and then slide object to produce an image.

NOTE: In both (a) and (b) the image was obtained by a combination of a slide and a reflection. This operation is known as a slide-reflection. The order in which the slide and reflection are performed is immaterial to the outcome (a slide-reflection is commutative).

(iii) Draw any polygon and perform a slide-reflection to obtain an image. A possible slide-reflection is shown in Diagram 3.

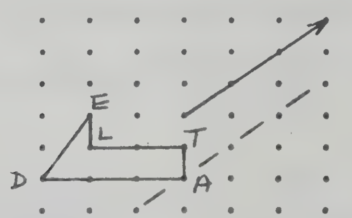


Diagram 3

Discuss congruence of the figure, and its image. e.g. $\overline{DE} \cong \overline{D'E'}$

$\angle TLE \cong \angle T'L'E'$

Traditional Exercises (TE)

These refer to the sets of traditional convergent type of problems given at the end of every lesson of the Program. Generally, Traditional Exercises have the following characteristics:

(1) Most of the problems were designed in the programmed instruction format, with gradual breaking down of the mathematical

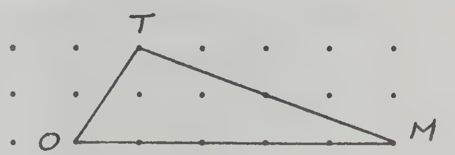
steps that would eventually lead students to the correct solution.

(2) Almost every problem was a conventional convergent type. For each problem or sub-problem, there was normally one and only one correct answer.

(3) For every lesson, there were usually more than two or three similar problems in different forms, which required direct and obvious application of the mathematical concepts or operations learned in that particular lesson. This was assumed to provide the necessary "drill" for students.

For the Traditional Unit on motion geometry, the 19 sets of Traditional Exercises (TE) given in the Program were adopted in their entirety. The complete set of 19 TE's with keys can be found in the Program (Edmonton Separate School Board, 1975). A few typical examples are given below.

(1) Copy $\triangle$TOM on your dot paper and draw the image for the slide (3R, 2U). Label image T'O'M'.



(a) What is the image of:

$\overline{TO}$ ________________

$\overline{OM}$ ________________

$\overline{MT}$ ________________

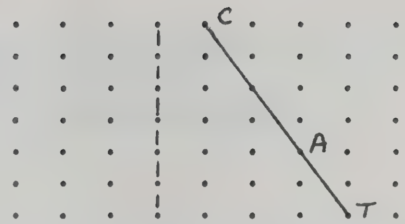
(b) What is the image of: point T ______

point O ______

point M ______

(c) Are the segments of $\triangle$TOM congruent to their images? __________

(2) Copy the figure below onto your dot paper and draw its reflection image. Label the image C'A'T'.



(a) $\overline{CA}$ and ______ are congruent.

$\overline{C'T'}$ and ______ are congruent.

$\overline{AT}$ and ______ are congruent.

(b) How many spaces are there from:

C to mirror line ______

C' to mirrow line ______

mirror line to A ______

mirror line to A' ______

T to mirror line ______

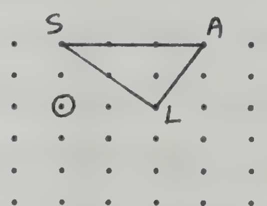
T' to mirror line ______

(c) Connect each named point and its image. The angle each segment makes with the mirror line is __________.

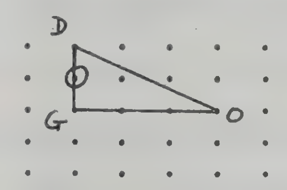
(d) What is true about $\overline{CC'}$, $\overline{AA'}$, and $\overline{TT'}$, when you look at all three segments together? ____________________

____

(3) Copy $\triangle$SAL and $\triangle$DOG on to your dot paper and draw the half-turn images. Label each image.



(a) Indicate pairs of congruent line segments for each triangle and its image.

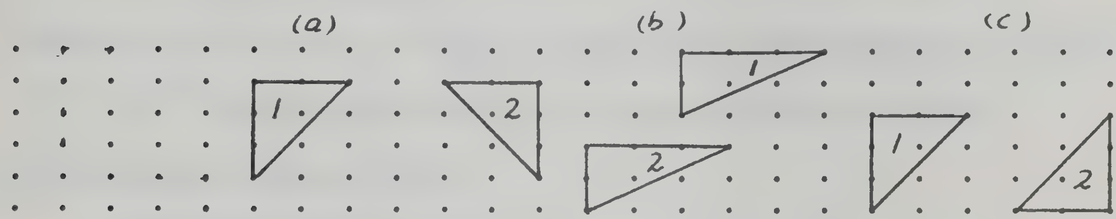


(b) $\angle$ SAL $\cong$ $\angle$ ______; $\angle$ ______ $\cong$ $\angle$ A'S'L'

$\angle$ GDO $\cong$ $\angle$ ______.

(c) $\angle$ DGO $\cong$ $\angle$ ______; $\angle$ ______ $\cong$ $\angle$ D'O'G'.

(4) Name the transformation which maps object 1 onto object 2 in each case below. Mark the correct symbols on the diagram to show this transformation. eg. ⟶ or ⊙ or - - - - - .



Inventive Exercises (IE)

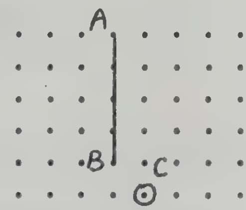
These refer to the sets of exercises especially designed for the Inventive Unit on motion geometry. They consist of two types of problem-situations: (1) the traditional convergent type which asks for a "single correct solution" to each problem, and (2) the inventive divergent type which emphasizes a "variety of solutions" to a given problem. Since the convergent type of problems are similar to those of the Traditional Exercises with some modifications to eliminate unnecessary routine "drill", only the inventive divergent type need therefore be discussed in this section. These inventive problems constitute about 60% of the total IE.

The main purposes of the IE were to encourage and reward divergent productions in solving mathematical problems. We have already shown that (1) the creative process or processes employed by a student to solve any creative problem cannot be identified, (2) the functioning of a creative process varies from problem to problem, and hence (3) it is more promising and practical to construct creative mathematical problem-situations in terms of the

three parameters: domain (D), operation (O), and range (R) (<u>supra</u>: 54ff). The following are a few examples to illustrate the various types of <u>inventive divergent</u> problems. (See Appendix A for the complete set of 19 sets of <u>Inventive Exercises and Suggested Solutions</u>.)

(1) <u>D and O are given, and a manifold solution (R) is elicited for a given problem</u>:

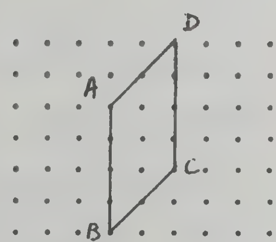
> Copy the figure on your dot paper and draw its ½-turn image. C is the turn centre. Label the image $\overline{A'B'}$.



> Join the points to their images and write down all the properties of ½-turns.

The D consists of the original segment $\overline{AB}$ and the turn centre C, with the ½-turn as the given O. The ½-turn image of $\overline{AB}$ seems to be the only element of the R. However, in this particular situation, R is really the set of "all the mathematical properties related to the ½-turn of a line segment." Similarly, the next problem asks for many elements of the R set:

> Copy parallelogram ABCD onto your dot paper. Draw in all the diagonals. Label the point of intersection O.
>
> List all the properties of the parallelogram.

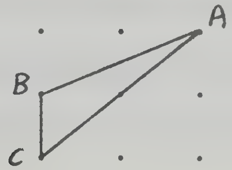


Other similar types of problems encourage students to produce many responses with given operations, such as,

> Enclose a region with 9 dots on your dot paper. Draw the following types of triangles with end-points (vertices) on those dots:
> (1) all possible acute $\triangle$s;
> (2) all possible obtuse $\triangle$s;
> (3) all possible right $\triangle$s.

Use a new 9-dot-region for each different triangle.

You are given a $\triangle$ABC within a 9-dot-region.

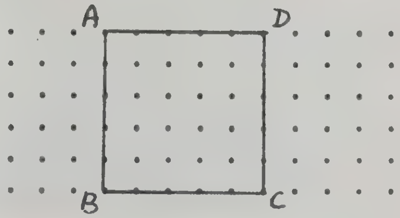


Use a new 9-dot-region for each different triangle, draw all the triangles that are congruent to $\triangle$ABC.

It can be seen that this type of inventive problem seems to combine many traditional convergent problems into a single problem. Nevertheless, students have to be sensitive to the various possible solutions, and probably also have to make conjectures and guesses about those triangles before actual constructions are performed.

(2) <u>D and R are given, and a manifold solution (O) is required</u>:

Find all possible single glides that produce images of ABCD which fall exactly onto ABCD itself.



The given square ABCD defines both the D and the R of this problem. The required O has been delimited to all the "single glides (or motions)" to define a "universe of discourse". To be creative in this situation implies the ability to recognize some or all of the following facts:

(a) There is only one trivial identity-slide.

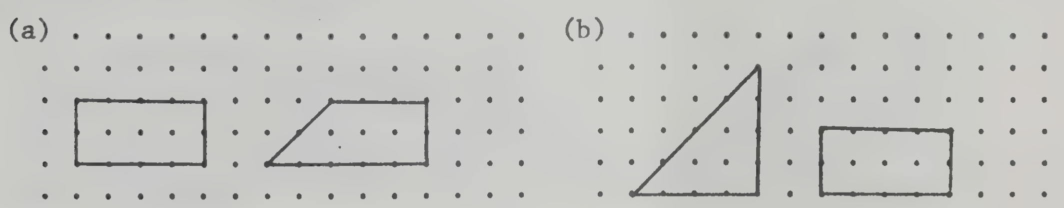
(b) Identity reflections are reflections in the lines of symmetry.

(c) A revolution of 360° about any given turn-centre is the only type of identity-turn.

(d) Since slide-reflection is defined as a "single glide" with line of reflection parallel to the slide arrow, the combinations of solutions in (a) and (b) are all the identity-slide-reflections.

The realization of all these facts requires the process of sensitivity to "see" the limitations and possibilities. The conclusions of (b) and (d) also imply the need to transform and redefine the situation in terms of identity-slides and lines of symmetry. A student who is aware of (c) and is able to redefine the problem into that of a simple problem of "locating turn-centre" can be said to exhibit the processes of mathematical creativity.

> Show how you can convince your friend that the following pairs of figures have the same area. (Perform a ½-turn on some part of one figure.)

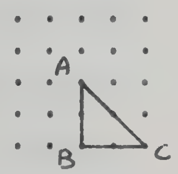


As these two problems were designed for the lesson on ½-turn, the operation defined is ½-turn. For each of the problems, one of the two figures is really the D, and the other, the R. The problems require the selection of appropriate O's that will transform figure D into figure R. Before any motion can be performed, students have to identify the appropriate part of figure D, and locate the appropriate turn-centre, such that through a ½-turn on that part, figure R can be obtained. Sensitivity is needed to select the

various possible parts.

(3) D is given and, R and O are to be found within some broadly defined "universe of discourse":

Form all possible reflection images of △ABC with vertices on dots within the 25-dot-region. Count these images:

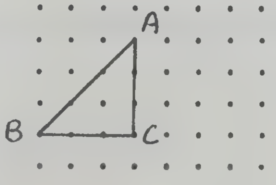


(a) There are ______ △s congruent to △ABC.

(b) There are ______ △s similar to △ABC.

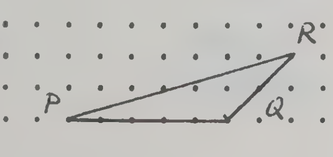
If students notice that "reflection images" can be images of △ABC formed through "successive reflections", they would probably "conjecture" that all possible triangles congruent to △ABC constitute the R set. If they further "see" that many congruent images of △ABC can be obtained through reflections in the sides of △ABC or its images, they have already solved the problem creatively. In fact, all the 36 triangles, including △ABC, define the R set of this problem.

Copy △ABC onto your dot paper. Locate your own turn center, such that the ½-turn image of △ABC and the original △ABC will form a parallelogram.

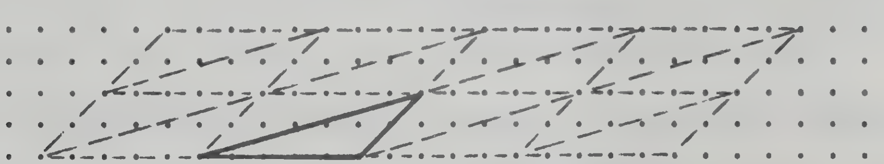


Though the R is not defined explicitly in this problem, there are only 3 possible parallelograms that can be formed. The realization of this limitation immediately helps to locate the 3 respective turn-centres for the solutions of the problem.

Using many different glides or combinations of glides form parallelograms with △PQR.



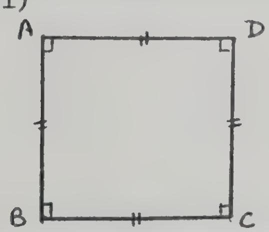
This seems to be a similar type of problem, but only asking for a larger solution set (O). However, closer scrutiny shows immediately that the R set of this problem actually contains infinitely many elements, since the only instruction is "to form parallelograms with $\triangle$ PQR", there is no restriction with regard to the number of images used. Therefore, besides the 3 obvious parallelograms similar to that of the previous problem, even number of triangles, i.e. $\triangle$ PQR and its images can be combined to form larger and larger parallelograms. For example,



With regard to the O's, there are also infinitely many possible ways. Nevertheless, to be creative in this situation seems to imply the flexibility of thinking that enables students to perform many different combinations of glides to create images of $\triangle$ PQR.

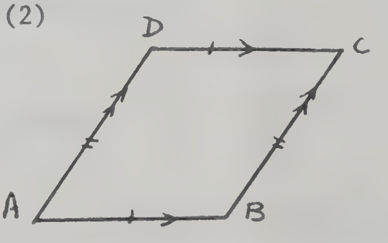
Draw the diagonals of the following figures and fill in the chart.

(1)



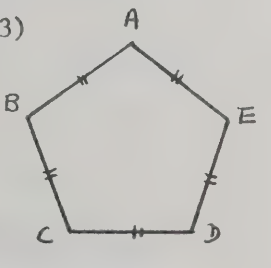
Square

(2)



Parallelogram

(3)



Regular Pentagon

	Number of Diagonals	Name all pairs of Congruent $\triangle$s	Rule used	Total number of pairs of $\cong$ $\triangle$s
(1)				
(2)				
(3)				

This is a combination of both traditional convergent as well as inventive divergent type of problem-situation. Though the O is to join two non-consecutive vertices of a figure to form a diagonal, the main purpose is to encourage divergent search of many solutions for the R set. For instance, the regular pentagon with its 5 diagonals gives rise to 6 different classes of congruent triangles, thus resulting in 95 pairs of congruent triangles.

As a whole, therefore the Inventive Exercises attempt to provide ample opportunity for the use of mathematical concepts and operations learned in more "flexible" and "open" problem-situations. At the same time, they prompt students to look for a "multitude" of solutions for any given problem. Traditional type convergent problems were also included to equip students with the prerequisite mathematical knowledge in motion geometry.

SUMMARY

The exploratory pilot study had significant implications for the development and evaluation of the Inventive Unit and Inventive Method designed for this study. Essentially, it demonstrated the effectiveness of the divergent-type of inventive problems in

enhancing students' mathematical creativity measured in terms of fluency, diversity and rarity. It furthermore clearly indicated the need to formalize the classroom instructional procedures to facilitate students' utilization of creative thinking processes and teachers' implementation of the inventive approach. This as well as other relevant implications were subsequently incorporated into the framework of the main study.

Consequently, two distinct instructional approaches were formulated: (1) the Traditional Method (TM) which consists of the Development of Concepts (DC) and the Traditional Discussion (TD) of the Traditional Exercises (TE), and (2) the Inventive Method (IM) which consists of the same DC and the Inventive Discussion (ID) of the Inventive Exercises (IE).

Correspondingly, two instructional units were constructed for the two methods: (1) the Traditional Unit, and (2) the Inventive Unit. The content domain of both units was delimited to the first 19 lessons of the Program. Both units contain a common part (the DC), and a unique part (respectively, the TE and the IE). The TE consists of the 19 sets of conventional convergent-type of exercises given in the Program. The IE consists of both convergent and divergent types of problems specially constructed for the IM. Finally, the construction of the 19 sets of IE were discussed in detail with the help of examples of inventive-divergent problem-situations.

CHAPTER V

METHODOLOGY AND IMPLEMENTATION

INTRODUCTION

The purpose of the present study is fourfold: (a) to justify the adaptation of an appropriate psychological model of creative problem solving in school mathematics, (b) to operationalize this model at the classroom level, (c) to develop special inventive instruction method and materials in a selected content domain of junior high school mathematics curriculum, and (d) to evaluate the effectiveness of the constructed method and materials. In previous chapters, we have elaborated on the psychological model adopted and the operationalization of this model, as well as the identification of appropriate content domain, and the eventual construction of relevant instructional procedures and materials. This chapter describes the evaluation phase of the study in terms of experimental design, implementation, research questions and hypotheses, and statistical procedures.

EXPERIMENTAL DESIGN OF THE STUDY

The research design of our study is an "evaluation design" which facilitates gathering of empirical data and "thereby making possible valid statements about the effects or outcomes of the

program, practice, or policy under study" (Airasian, 1974: 159). Two basic questions are crucial to such an evaluation study (Campbell and Stanley, 1973: 5):

(1) Internal Validity: This refers to the necessity to demonstrate that the experimental treatments did make a difference in a specific experimental instance. The determination that the program under study did produce the result observed is "the basic minimum prerequisite for the design of interpretable studies" (Airasian, 1974: 162). In order to be able to attribute the obtained effect confidently to the independent variable manipulated by the investigator, the orthodox Nonequivalent Control Group Design was adopted. As Campbell and Stanley (1973: 47-50) argue, this design neatly controls for the rival plausible hypotheses derived from such extraneous variables as history, maturation, testing, instrumentation, mortality and selection. Furthermore, two additional factors helped to strengthen the internal validity of our study:

(a) The recent introduction of "motion geometry" to the junior high school mathematics curriculum in the Edmonton Separate School System provides us with a naturalistic and realistic situation for conducting research. This "realism" aspect helps to control for experimental errors due to "novelty of situation".

(b) The experimental and control groups of the study, though under different instructional methods (IM and TM), did have a similar Development of Concepts (DC) phase to ensure the equivalent exposure to "content" instruction for both groups. This "experimental control" helps to eliminate other rival hypotheses

which would attribute any difference in students' ultimate achievement to factors other than the effects of the "inventive component" of the IM treatment, such as "difference in content learned".

(2) External Validity: This refers to the question of generalizability of the results of an experimental study: "to what populations, settings, treatment variables, and measurement variables can this effect be generalized" (Campbell and Stanley, 1973: 5). Since this study is the first attempt to operationalize Boychuk's (1974) creative problem solving model in school mathematics learning, its results should be regarded as useful primarily for providing relevant information for further, similar large-scale studies. Thus, while the question of external validity may have some relevance for our design, it can be safely ignored at this stage of creativity research.

In the following discussion, the "pretest-posttest control group design" will be specified in terms of the student sample, the teacher, the treatments and instrumentation.

Student Sample

Participants in the experiment were two grade eight classes in St. Edmund Separate School, Edmonton. There were 30 students in the experimental group and 28 in the control group. Though inflexibility of the school schedule prevented random assignment of individuals to treatment and control groups, intact classes were

assigned randomly to one of the two groups. After eliminating students who were absent from one or more of the tests, complete data were found to be obtained for 41 students: 20 from the experimental group and 21 from the control group. Hence the final sample consists of these 41 students.

Teacher

The two classes were taught by the same regular mathematics teacher, who holds a B.Ed. degree in Science and Mathematics teaching, and prior to the experiment had one year experience in teaching junior high school science. He administered all the tests to the students. To ensure that the treatments were carried out as prescribed, and to provide relevant classroom information for the teacher, the investigator undertook regular classroom observations over the experimental period. The *Observer Rating Scale of Teacher Behavior* developed by Naciuk (1968) was employed to collect reliable data on the classroom implementation of the two treatments. A description and discussion of this instrument will be given in Chapter VI.

Treatments

The two treatments were (1) *Inventive Method (IM)*, and (2) *Traditional Method (TM)*. The "experimental group" was taught through the IM with the *Inventive Unit*, and the "control group"

followed the Traditional Unit through TM. These methods and units have already been described in detail in earlier chapters. The treatments were conducted over a period of 6 weeks with a total of 30 instructions, each lasting 36 or 37 minutes.

Instrumentation

In order to determine the relative merits of the two instructional methods, it was deemed necessary to gather as much information as possible of the mathematical background of the participating students. Three scores were obtained from students' school records prior to the experiment:

(a) Grade 7 Mathematics Achievement Score (GR7).

(b) Grade Point Average of 5 Tests (A5T): This grade appeared on students' grade eight Report Cards. The 5 tests covered the following topics -- revision of computational skills, rational numbers, set theory and metric system.

(c) Metric Area Test Score (MAR): This is the major test immediately prior to the treatments. The test covered metric units for area measurements, as well as simple area calculation limited to square and rectangle. This was separated from A5T because of its close relation to some of the pretests and posttests employed for this investigation.

Besides these three scores on students' records, 6 different tests specially designed or relevant to our purposes were administered to all the students:

(1) Piagetian Sectioning Test (SEC): This is a 32-item Pretest requiring students to perform sectioning tasks on given solids, and to identify the plane figures formed by the intersection of the cutting plane with the geometric solid. The test consists of two parts: (a) Drawing, and (b) Multiple-Choice, each containing 16 items.

(2) Traditional Motion Geometry Test (TMG): This is a 40-item multiple-choice Posttest which assesses students' understanding of basic concepts and operations in motion geometry covered in the 19 lessons.

(3) Creative Geometry Test I (CG1): This is a 2-item Pretest, designed to assess students' level of mathematical creativity in general geometrical problem-situations prior to the treatments.

(4) Creative Motion Geometry Test (CMG): This is a 2-item Posttest designed to assess students' ability to solve problem-situations in motion geometry creatively.

(5) Creative Geometry Test II (CG2): This is a 2-item Posttest designed to assess students' level of mathematical creativity in general geometrical problem-situations at the end of the treatments.

(6) Transfer Area Test (TAR): This is a 14-item Posttest designed to assess students' ability to transfer the learned knowledge in motion geometry to traditional area-finding problem-situations.

The construction of these 6 test instruments and the related problems of validity and reliability will be discussed in greater

detail in the next chapter. Complete test instruments are included in the Appendices.

IMPLEMENTATION OF THE TREATMENTS

In our study, a single teacher was required to teach two grade eight classes in two different instructional methods: the IM and the TM. Obviously, no plausible conclusions could be drawn from the study if the teacher in fact taught the two instructional units in more or less the same way. There is hence a need to ensure the faithful implementation of the treatments as prescribed. Four procedures were employed for this purpose: (1) orientation prior to the experiment, (2) classroom observation, (3) discussion with the teacher, and (4) quantitative analyses of teacher's classroom behavior.

The orientation and discussion formed the on-going inservice procedures which helped to familiarize the teacher with the theoretical framework as well as the operational procedures of the study. They also allowed the investigator to monitor implementation of the two treatments, such that the effects of the treatments were not contaminated by misunderstandings on the part of the teacher.

Orientation and Discussion Sessions

There were two orientation sessions before the treatments.

The first session lasted about forty-five minutes, during which the main purposes, the conceptual and operational definitions of creativity, the instructional methods and materials were

discussed with the teacher.

The second session lasted about an hour, during which the teacher was given all the needed materials: the pretest problem sheets, the Inventive Unit Booklet with suggested solutions, the Traditional Unit Booklet with key to exercises, and the 19 sets of Inventive Exercises for the experimental group. Since the main difference between the IM and the TM of instruction falls into the "control function" of classroom instruction, the following four guidelines were emphasized during discussion with the teacher:

(a) For both IM and TM, the teacher should follow closely the Development of Concepts suggested in the Program (Edmonton Separate School Board, 1975) in developing the 19 lessons.

(b) For every lesson of each group (experimental or control), the first half of the class time, about 15 to 20 minutes, should be spent on classroom discussion of exercises (IE or TE) assigned in the previous lesson. This is to give feedback for assigned exercises, and guide students' thinking along the line of training designed for the particular group.

(c) Whenever explanations, hints or prompting are needed to guide students in solving those exercises (IE or TE), the information should be given according to the key or special suggested solutions provided for the respective groups.

(d) Since the IE were creative problem-situations specially constructed to facilitate and stimulate the functioning of the creative processes of sensitivity, redefinition and conjecturing, the teacher was encouraged to base his classroom discussion (ID) on

these three processes. Specifically, he should elicit inventive divergent solutions from students by (i) leading them to "see" deficiencies and shortcomings inherent in a problem-situation, thus opening up new possibilities, (ii) prompting them to redefine the problems in terms of mathematical concepts or operations learned, or (iii) encouraging them to make guesses and try out new solutions.

During the orientation and discussion sessions, questions raised by the participating teacher were answered and clarified. These four guidelines also served as criteria for classroom observation during the monitoring of implementation of the two instructional methods. Whenever this investigator deemed it necessary, a short discussion session was carried out after the lesson with the teacher regarding needed adjustments to ensure faithful implementation of both instructional methods according to our design.

Classroom Observation

During the six-week period, the investigator was present in 12 of the 30 periods of each group to (a) ensure that the two instructional methods were implemented as prescribed, and (b) correct any deviation from the designed procedures when necessary. The four main guidelines mentioned above were employed to categorize the teacher's classroom behavior as appropriate or inadequate for the respective instructional methods (IM or TM).

Furthermore, the investigator also rated the teacher's instructions in both groups on the <u>Observer Rating Scale of Teacher</u>

Behavior (Naciuk, 1968) to obtain quantitative data about teacher's classroom behavior; such data provided a basis for critical appraisal of the implementation of the prescribed instructional methods.

RESEARCH QUESTIONS AND HYPOTHESES

As indicated earlier, we utilized an "evaluation design" to collect empirical data which will facilitate a critical assessment of our instructional method and materials. This experimental design is basically the "nonequivalent control group design". To obtain valid and reliable conclusions about the effects of our Inventive Method of teaching grade eight motion geometry, three types of behaviors were observed and analysed:

(1) Teacher's Classroom Behavior: Information about the classroom teaching was collected to ensure that the two instructional methods were implemented as planned.

(2) Students' Entrance Behavior: Relevant background information of all students involved in the present study was gathered and examined to decide whether any difference exists between the two groups with respect to related mathematical or psychological variables.

(3) Students' Terminal Behavior: The outcomes of the two instructional methods were ultimately evaluated in terms of some selected mathematical or psychological variables.

The information collected with respect to these three types

of behaviors would answer the following research questions which are further stated in the form of null hypotheses.

Teacher's Classroom Behavior

Since only one teacher was involved in the study, it was quite possible for him to teach the two groups of students in essentially the same approach. The main purpose of the two different methods of instruction was to provide two rather different types of learning experiences to two groups of students, thus enabling us to compare the effectiveness of the IM. One obvious way of ensuring the faithful implementation of the designed instructional units and methods is, of course, to obtain information and data on the teaching methods through direct observations of what goes on in the classroom (Naciuk, 1968: 50). As Rosenshine (1970: 280) concluded from his observation on curriculum evaluation, the lack of information on classroom interaction hinders evaluation of different curricula or instructional methods.

We therefore undertook regular classroom observation to obtain quantitative information about the teaching and learning activities in the two groups. For this purpose, the Observer Rating Scale of Teacher Behavior (Naciuk, 1968) was used by this investigator while observing the teacher in action during those classroom observations. This Rating Scale rates teacher's classroom behavior on six dimensions which appear to differentiate the IM from the TM of instruction. Equal numbers of ratings were made on the Development of Concepts

(DC) phase and the Discussion (ID or TD) of both groups. The results of these ratings provide therefore a basis for answering the following pertinent research question:

"Do the two instructional methods differ on the Observer Rating Scale with respect to the two phases of instruction: Development of Concepts and Discussion (ID and TD)?"

This may be stated in null hypothesis as follows:

Null Hypothesis 1:

There is no significant difference, (a) between the two instructional methods with respect to the Development and Discussion phases of instruction, and (b) between these two phases within each method.

Students' Entrance Behavior

The learning of any mathematical topic is dependent upon prior mathematical knowledge. Though we have claimed that learning of motion geometry topics selected for our study does not demand the mastery of any formal prerequisite mathematics, it is reasonable to suggest that prior mathematical experience would affect the learning of our selected subject content. To investigate this question, students' achievement scores in mathematics prior to the experiment are therefore needed. Three traditional achievement scores were obtained for students involved in the study: (a) Grade 7 Mathematics Achievement Score (GR7), (b) Grade Point Average of 5 Tests (A5T), and (c) Metric Area Test Score (MAR). The materials covered by

these tests appeared highly relevant to our investigation (supra:114).

In addition to subject content, mathematics learning also requires a certain mental capacity. The main purpose of our study is to affect students' creative thinking capacity through the inventive approach of instruction. There is therefore a need to assess students' levels of mathematical creativity prior to the experiment to see whether any initial difference exists between the two groups. For this purpose, scores on the Creative Geometry Test I (CG1) were obtained.

Students of the two groups were taught some basic concepts of geometric transformation via two different instructional approaches. Undoubtedly, their spatial ability would be likely to affect their learning. Piaget and Inhelder (1967, 1969) have concluded from their research that the conception of space of children develops through three levels: (a) topological, (b) projective, and (c) Euclidean. A child arrives at the Euclidean level of geometric maturity at the age of approximately twelve years, when she/he is able to respond correctly to the "Euclidean Sectioning tasks". The important implication of Piaget's developmental theory is that geometry instruction should take into consideration the mental readiness or maturity of the students. Furthermore, motion geometry in this study was essentially the transformational treatment of Euclidean geometry. It is therefore necessary to assess the level of geometric maturity of all students involved in the experiment. The purpose, however, was not to incorporate these various levels of geometric

perception defined by Piaget into our instructional procedures, but to compare the mental readiness of students under the two instructional methods. For this purpose, students' achievement scores on Piagetian Sectioning Test (SEC) were obtained.

In effect, the relevant research question pertaining is as follows:

"Do students of the two groups differ with respect to their scores on these 5 pretests?"

Stated in null hypothesis, we have

Null Hypothesis 2:

There is no significant difference between the experimental and control groups with respect to students' scores on the 5 pretests: GR7, A5T, MAR, SEC and CG1.

Students' Terminal Behavior

The learning of content matter is a natural objective of any mathematics instruction. How well did students under the two treatments learn the topics of motion geometry presented to them in different methods of instruction? Since the subject content was taught in two different methods, the IM and the TM, two tests were constructed: (a) Traditional Motion Geometry Test (TMG) to assess students' performance on conventional convergent type of problems, and (b) Creative Motion Geometry Test (CMG) to assess students' performance on inventive divergent types of problems.

In addition, since "the whole idea of education is

predicated on transfer of training" (Bourne, Ekstrand and Dominowski, 1971: 144), and "problem solving" is "a form of transfer of learning in which experience in one task influences performance on another task" (DeCecco, 1968: 439), the transferability of creative problem solving experience warrants serious consideration. DeCecco (1968: 442) further argues that "the measurement of how much learning has occurred in problem solving is the student's ability to solve new problems of the same class." The two instructional methods (IM and TM) basically emphasize the problem solving aspect of motion geometry through 19 sets of Exercises (IE and TE) and the Discussion (ID and TD). We would therefore like to see whether students of both groups are able to transfer their problem solving experience in the area of motion geometry to other related geometrical problem-situations. "New problems of the same class" means that transfer of learning from one situation to a different situation occurs only when there are some common characteristics and attributes between the two situations (Osborne, 1976: 12). Obviously, such new situations conducive to transfer of the kind of learning students experienced in our study should be geometrical. Since the Inventive method and the related Inventive Exercises were designed to enhance students' creative problem solving ability in mathematical situations, the Creative Geometry Test II (CG2) was constructed. CG2 assesses students' ability to transfer creative problem solving processes to "open" geometrical problem-situations which require inventive divergent solutions. Furthermore, given that the topic immediately prior to our experiment was on metric area units with simple calculation of

areas of squares and rectangles, it was meaningful to construct the Transfer Area Test (TAR). TAR presents students with traditional convergent type of area-finding problem-situations, which can be solved easily by applying some basic notions of motion geometry to the problems. The purpose was to assess students' ability to transfer the basic concepts of geometric transformation learned to facilitate the computation of areas of geometrical figures. With regard to students' terminal behavior, the investigator was thus primarily concerned with the following question:

"Do students of the two groups differ with respect to their scores on these 4 posttests?"

In null hypothesis form, this becomes:

Null Hypothesis 3:

There is no significant difference between the experimental and control groups with respect to students' scores on the 4 posttests: TMG, CMG, CG2, and TAR.

These three null hypotheses which are designed to answer the research questions about teacher's classroom behavior, students' entrance and terminal behavior, will be further analysed in detail in Chapter VII.

STATISTICAL PROCEDURES

As Treffinger, Renzulli and Feldhusen (1971: 105) have stated clearly, one of the basic underlying assumptions of studies attempting to measure creative thinking is that the "creative

process is complex, or multidimensional, in nature." Because of such inherent complexity of creative thinking phenomena, there are (a) a need for broadening the selection of test tasks as measuring instruments, and (b) a need to utilize complex multivariate statistical procedures to analyse the data obtained (Treffinger and Poggio, 1972: 257). In our study, multiple observations were therefore made on the students involved through 9 different tests, and the multivariate method employed to analyse most of the data obtained.[1] Cramer and Bock (1966: 604) have stated succinctly the advantages of such a statistical method:

> Multivariate analysis provides models and procedures for dealing separately with each of a number of variables in estimation, while at the same time providing tests of hypotheses which lead to a single probability statement referring to all variables jointly.

Instead of means on individual variables (tests), multivariate techniques analyse "vectors of means", where each element of the vector is a group's mean for a particular variable (test). Each group's vector of mean scores is called the centroid of that group. We will now discuss three major multivariate procedures utilized to compare the various "centroids" of the two treatment groups: (1) Hotelling's T^2 Test, (2) Multivariate Analysis of Variance (MANOVA), and (3) Multivariate Analysis of Covariance (MANCOVA).

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[1] Univariate t-test, analysis of variance (ANOVA) or analysis of covariance (ANCOVA) were employed on a few occasions.

Hotelling's T^2 Test

Scores of students' performance on the 5 pretests (GR7, A5T, MAR, SEC and CG1) were obtained prior to the two treatments. These test scores provide a basis for deciding whether students of the two groups differ significantly. Since the tests are mathematical in content, scores on different tests are probably correlated to some extent. The univariate t-test on each test score does not utilize this correlation among the tests, and hence cannot provide us with an overall picture of differences between the two groups (Timm, 1975: 216). Similarly, since observations on teacher's classroom behavior were rated on 6 dimensions on the Observer Rating Scale, the 6 ratings on each classroom observation would be intercorrelated. In such cases where some p measures (variables) are involved in the comparison of two groups,

> Hotelling's T^2 provides a means of testing the overall null hypothesis that the two populations from which the two groups were sampled do not differ in their means on any of the p measures (Harris, 1975: 13).

Consequently, Two-Sample Hotelling's T^2 Tests were conducted on scores of all the 5 pretests, and on the ratings on the Observer Rating Scale.

Hotelling's T^2 Test is a direct generalization of the Student's t-test to situations involving more than one dependent variable (Harris, 1975: 67). Suppose we have two independent random samples of observations on some p measures (variables), which have a multivariate normal distribution, with a common but unknown

"dispersion" or "covariance" matrix of full rank p. If μ_1 and μ_2 are the respective "centroids" of the two populations from which the two samples are obtained, then the statistic T^2 tests the null hypothesis:

$$H_0: \mu_1 = \mu_2 \quad ,$$

that the population centroids are identical, against the alternative hypothesis:

$$H_1: \mu_1 \neq \mu_2 \quad ,$$

of different centroids (Morrison, 1976: 137).

Multivariate Analysis of Variance (MANOVA)

The effects of the two treatments in the present study were measured in terms of students' terminal behavior (performance) on 4 posttests: TMG, CMG, CG2 and TAR. Each of the two creative tests (CMG and CG2) was scored on three dimensions: fluency, diversity and rarity, all of which were derived from the same set of responses (solutions) of each student, and hence were inter-correlated. Moreover, the 4 tests being mathematical tests based on junior high school geometry, would probably be intercorrelated to some extent. It is therefore reasonable to study all or groups of these variables (tests) simultaneously rather than individually, i.e. through the procedures of MANOVA. The strength of MANOVA has been demonstrated by Kerlinger and Pedhazur (1973: 355-358) in a hypothetical example. They constructed a case of 3 groups and 2 dependent variables, where univariate ANOVA detects no significant differences on either

variable, but MANOVA shows significant differences when both variables are analysed simultaneously.

MANOVA, like the univariate ANOVA, focuses upon differences between group centroids which reflect the systematic differences in performance between students of the two treatment groups. The procedure employed for our analysis was the One-Way MANOVA, Fixed Effect Model, where the design matrix assumes that the elements of the parameter matrix are to be constant. Such "fixed effect design" is sometimes called Type I Model (Bay, 1969: 13; Winer, 1971: 163). Suppose p criterion variables (posttests) are observed on each of the N students in our study, then the multivariate linear model under the multivariate Gauss-Markoff setup has the form (Timm, 1975: 185):

$$\underset{(N \times p)}{Y} = \underset{(N \times q)}{X} \; \underset{(q \times p)}{B} + \underset{(N \times p)}{E}$$

where, Y = observation matrix on the p criterion variables for the N students;

X = known design matrix of rank $r \leq q \leq N$;

B = effect matrix with q nonrandom parameters for each response; and

E = matrix of random error.

This model is based on three assumptions: (a) independence of observations, (b) homogeneity of dispersion matrix, and (c) multivariate normal distribution (Bay, 1969: 33). Two significance tests are thus usually performed in MANOVA (Amick and Walberg, 1975: 226):

(a) A test of group differences to determine whether the two group centroids are significantly different or whether their

differences occurred by chance, in a multidimensional space. The null hypothesis is:

$$H_{o1}: \mu_1 = \mu_2 \text{, against } H_{11}: \mu_1 \neq \mu_2 ;$$

where μ's are population centroids for the respective groups.

(b) This test of equal centroids is based on the assumptions of homoscedasticity and normality. The latter can be justified partly by our assumption about the distribution of mathematical creativity among student population, and partly by using the central limit theorem (Cooley and Lohnes, 1971: 38). Hence only the former assumption needs investigation to test the null hypothesis of the equality of the two group dispersion matrices:

$$H_{o2}: \Sigma_1 = \Sigma_2 \text{, against } H_{12}: \Sigma_1 \neq \Sigma_2 ;$$

where Σ's are the respective population dispersions.

Multivariate Analysis of Covariance (MANCOVA)

The precision of our experimental design can be increased by controlling for variability due to experimental error. There are two general methods for such purposes: direct and statistical controls (Winer, 1969: 752). Direct control focuses on means of improving the experimental designs and increasing the accuracy of the measurements. To some extent, these have been achieved through our selected and constructed experimental design and test instruments. Statistical control attempts to utilize more empirical information and to remove potential sources of bias in the experiment. In this regard, we have chosen multivariate statistical procedures to

consider many variables simultaneously. We also employed MANCOVA to eliminate possible sample biases. A major source of such biases was the variability in students' entrance behavior (performance) on the pretests. Since these tests are mathematical and psychological in nature, they are likely to have some relation with students' terminal performance. In other words, students' initial competency in mathematics and initial level of geometric maturity and mathematical creativity are likely to contribute to their final achievement in traditional as well as creative mathematical tests. We would therefore like to eliminate that part of students' ultimate achievement due to initial competency and capacity, and thus attribute the remaining part to the effects of different treatments. The technique of MANCOVA serves to achieve this sort of statistical control by estimating the group differences on posttest scores after they have been adjusted for initial differences in the pretest scores (Bock, 1966: 829). MANCOVA eliminates the differences of treatments caused by students' entrance behavior both from within and between groups and tests for the "pure effects" of the treatments with greater efficiency (Rao, 1973: 288).

Using the same notations as that of MANOVA above, and under the <u>multivariate Gauss-Markoff setup</u>, the model for MANCOVA is (Timm, 1975: 479):

$$\underset{(N\times p)}{Y} = \underset{[N\times(q+h)]}{[X, Z]} \underset{[(q+h)\times p]}{\begin{bmatrix} B \\ \Gamma \end{bmatrix}} + \underset{(N\times p)}{E}$$

where the number of covariates (pretests) is h, and

$\mathcal{Z}$ = observation matrix on the h covariates for the N students; and

Γ = matrix of regression weights.

The validity of MANCOVA, as well as ANCOVA depends upon the assumptions of (a) randomization, (b) independency, (c) normality, (d) homogeneity of regression, and (e) homogeneity of variances (Elashoff, 1969). Since treatments were assigned to intact groups at random in the present study, and covariates were measured prior to treatments, the first two assumptions are more or less tenable. Normality is assumed for the same reasons given for MANOVA in the previous section. We therefore need to examine the last two assumptions as part of our MANCOVA procedures. Three significance tests will be considered:

(a) Homogeneity of Regression: MANCOVA assumes that the regression planes of the criterion scores (posttest scores) on the covariates (pretest scores) for the two groups are parallel. This parallelism hypothesis implies that there is no "treatment-slope interaction". If Γ_1 and Γ_2 are the two matrices of regression weights for the two groups, the null hypothesis of this parallelism test is as follows (Timm, 1975: 346):

$$H_{01}: \quad \Gamma_1 = \Gamma_2, \text{ against } H_{11}: \Gamma_1 \neq \Gamma_2.$$

(b) Homogeneity of Variances: MANCOVA procedures also assumes that the residual variation about the regression planes for the two groups are homogeneous. In testing this hypothesis of homoscedasticity,

$$H_{02}: \Sigma_1 = \Sigma_2, \text{ against } H_{12}: \Sigma_1 \neq \Sigma_2,$$

where Σ's are the respective population dispersion matrices.

(c) Differences of Adjusted Centroids: If the first two null hypotheses are tenable, a test for differences between group centroids, eliminating the linear association of criterion scores on covariate scores, can then be made (Timm, 1975: 489):

$$H_{03}: \mu_1 = \mu_2, \text{ against } H_{13}: \mu_1 \neq \mu_2,$$

where μ's are the respective population "adjusted" centroids.

SUMMARY

The evaluation phase of the study employed a Nonequivalent Control Group Design to obtain empirical data for assessing the effectiveness of the constructed Inventive Method (IM) of teaching grade eight motion geometry. To ensure faithful and satisfactory implementation of the two instructional methods (IM and TM), orientation sessions were conducted for the participating teacher prior to the experiment, and regular classroom observations were undertaken by the investigator to gather information about classroom activities of both groups and to monitor the implementation of the constructed instructional methods and materials.

Furthermore, quantitative data were collected about the teacher's classroom behavior using the Observer Rating Scale. Students' entrance as well as terminal behaviors (performance) were measured by tests selected or specially constructed, in order to facilitate critical assessment of the IM and TM of instruction. These test instruments included 5 pretests (GR7, A5T, MAR, SEC and

CG1) and 4 posttests (TMG, CMG, CG2 and TAR).

The data thus obtained on students' and teacher's behaviors provide the basis for answering three fundamental research questions of our study:

(1) Were the treatments, IM and TM, implemented as designed?

(2) Did the students of the two groups (experimental and control) differ initially in terms of their mathematical background and related mental capacity?

(3) Did the students of the two groups perform differently at the end of the treatments?

To test the hypotheses derived from these three questions, multivariate statistical techniques including Hotelling's T^2 Test, MANOVA and MANCOVA were selected as the appropriate analytical tools for most of the analyses, and univariate t-test, ANOVA, and ANCOVA were selected for a few occasions.

CHAPTER VI

CONSTRUCTION AND VALIDATION OF MEASURING INSTRUMENTS

INTRODUCTION

The discussion thus far has focussed on the selection of a theoretical model and an operational definition of mathematical creativity, and deriving from these conceptual frameworks practical guidelines for the construction of an inventive instructional method and materials. Our next important concern is to construct and validate relevant measuring instruments to assess the effectiveness of the designed IM and TM of instruction. A total of ten instruments[1] were selected: five pretests (GR7, A5T, MAR, SEC and CG1), four posttests (TMG, CMG, CG2 and TAR), and a classroom observation instrument (Observer Rating Scale). The rationale for using these instruments has already been provided in the previous chapter, and a thorough discussion of the theoretical principles underlying the construction of the three creative tests (CG1, CMG and CG2) given in Chapter III. We present in this chapter the final forms of the constructed tests, the scoring procedures for the creative tests, and the related problems of validity and reliability of the various

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[1]The tests CG1, CMG, CG2 and TAR were specially constructed by this investigator.

measuring instruments.

THE CREATIVE TESTS

Guidelines for Creative Problem-Situations

The main underlying principles for the construction of creative problem-situations in school mathematics can be stated briefly in terms of the following practical guidelines[2]:

(1) In a creative mathematical problem-situation, one or two of the three parameters, D (domain), O (operation) and R (range), are given, and students are required to search for solution set(s) of the remaining parameter(s).

(2) At least one of the given parameters is stated in quite broad terms with many possible interpretations and implications embedded.

(3) A variety of solutions or solution-sets are required for the unknown parameter(s).

(4) Flexibility is the key feature of the problem-situation, although ambiguity should be avoided by explicitly delimiting a universe of discourse.

The three creative tests constructed for the present study will be discussed in terms of these four guidelines.

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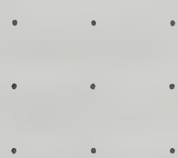
[2]Details of the conceptual foundations of these guidelines can be found in Chapter III, while the pilot study related to these points is detailed in Chapter IV.

Creative Geometry Test I (CG1)

Due to the "open-ness" of creative problem-situations, the need to allow optimal time for students to indulge in inventive divergent thinking, and the vast number of tests involved in our study, a carefully designed two-item test that could be completed in a single period of about 36 to 37 minutes, was deemed adequate for our purposes.

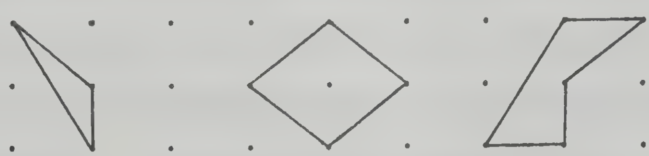
The following two problems were developed for the pretest CG1:

(1) You are given a region with 9 dots:



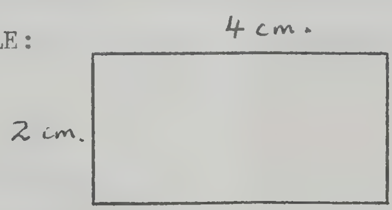
You are asked to draw as MANY DIFFERENT closed geometrical figures as you can think of. The end-points (vertices) of your figures must lie on the dots.

EXAMPLES:



DO NOT DRAW ALL YOUR FIGURES ON ONE REGION!
USE A NEW 9-DOT-REGION FOR EACH DIFFERENT FIGURE!!

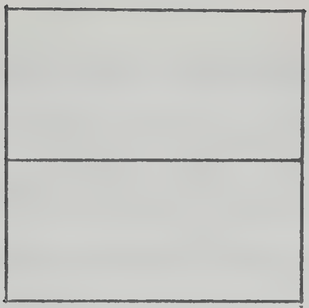
(2) This is a RECTANGLE:



You are asked to say something about this rectangle by doing something about it.

EXAMPLES:

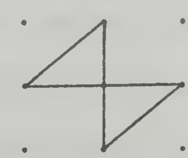
(a) A square can be formed by putting 2 such rectangles together:



(b) The rectangle can be separated into a 3-sided figure (triangle) and a 4-sided figure (quadrilateral):

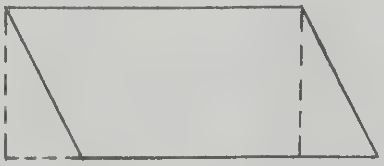
Now go on to write as MANY IDEAS as you can about this rectangle. DRAW A FIGURE FOR EACH STATEMENT YOU PUT DOWN!!

For problem (1), domain D consists of "a square region of 9 dots (3x3)." The required operation O is "to select appropriate sets of points and join consecutive pairs of them" to form closed geometrical figures which make up the range set R. Undoubtedly, D is clearly specified, and R consists of a multitude of solutions. O in this situation is partially defined through the three illustrations to mean the forming of closed figures by joining pairs of points. However, sensitivity and flexibility in thinking are called upon to notice the "freedom" embedded: (a) a zigzag line or a curve can be formed by joining a pair of points, and (b) a closed geometrical figure can consist of more than one part joined together, i.e. a figure with vertices of degree greater than 2, such as:



Problem (2) is a more directed type of conjecturing and sensitivity problem-situation where redefinition is implied. D is

the given rectangle. O and R though given very broadly, are nevertheless delimited to two possible operations: (a) operation on the whole rectangle, and (b) operation within a single rectangle. A square was formed by joining two such rectangles in the first example, whereas the second example showed how different polygons can be formed by cutting the rectangle with a line segment. In this situation, students have to select their own operations and look for the admissible solutions of R. The "universe of discourse" delimits the permissible O and R sets, both of which must be mathematical. Such responses as forming a "house" or a "tank" are clearly unacceptable. Students' sensitivity is provoked to look for various formation of rectangles by "tiling", and different ways of cutting with line segment(s) or curve(s). Furthermore, the process of redefinition is needed to combine the two kinds of operations. One can perform cutting on the square formed by joining two rectangles, or recombine the various separated parts of a rectangle to form an entirely new geometrical figure. For example, a parallelogram can be formed by sliding the triangle of example (b) to the right side:



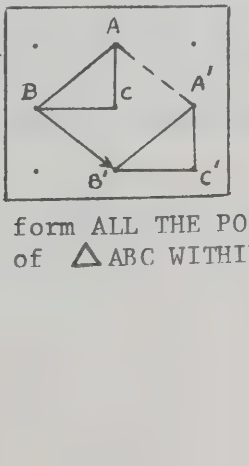
These two problems therefore appear to reflect the three creative processes of sensitivity, redefinition and conjecturing. Moreover, these problem-situations do not require special mathematical content knowledge other than the ability to draw simple geometrical closed figures. This two-item <u>Creative</u>

Geometry Test I (CG1) was thus regarded as an adequate measuring instrument for assessing students' initial level of mathematical creativity in general geometrical problem-situations.

Creative Motion Geometry Test (CMG)

We have argued earlier that creative problem solving demands the accessibility of pre-requisite mathematical knowledge. In effect, it is necessary to construct creative motion geometry problem-situations which would require concepts and operations in the 19 lessons that most students are likely to have mastered. Among the many notions of geometric transformation learned, the four basic motions (slide, reflection, turn and slide-reflection) were repeated quite often throughout the instruction of the IM and the TM. It would then be reasonable to assume that by the end of the 6-week period, all students had learned these four essential concepts and operations to enable them to tackle the following two motion geometry problems creatively:

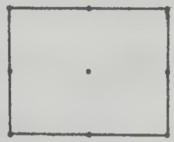
(1) You are given a closed region with 9 dots and $\triangle$ABC.
You can form MANY IMAGES of $\triangle$ABC through a slide, or a turn, or a reflection, or a slide-reflection, or a combination of glides.
EXAMPLE:



You are asked to form ALL THE POSSIBLE DIFFERENT IMAGES of $\triangle$ABC WITHIN THE 9-DOT-REGION.

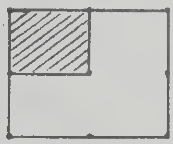
DO NOT DRAW ALL YOUR IMAGES ON ONE REGION.
USE A NEW 9-DOT-REGION FOR EACH DIFFERENT IMAGE.
Indicate your motion with appropriate notation: --------, ⟶ , or ↻ .

(2) You are given a SQUARE on a 9-DOT-REGION:

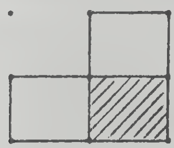


By selecting your own motion, you can change the square into some other polygon.

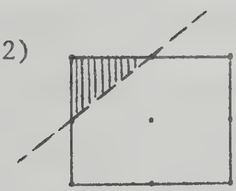
EXAMPLES: (1)



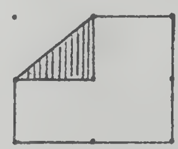
Slide shaded part (1R,1D): new polygon:



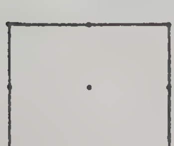
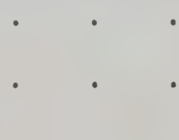
(2)



Reflect shaded part in the mirror line: new polygon:



Now, select your own SHADED PART, and MOTION (slide, turn, reflection, slide-reflection, or combination of glides).
Write down all your different results in the following chart:

SHADED PART	MOTION	RESULTING POLYGON
		

In problem (1), D consists of a 9-dot region with a given $\triangle$ABC. Students are asked to select their own O (motions or glides) to obtain appropriate elements of the R, set of images of $\triangle$ABC. As demonstrated in Chapter III (supra: 62), this is indeed a creative problem-situation that calls upon the functioning of sensitivity, redefinition and conjecturing.

For problem (2), the square with 9 dots defines the D. Students have to select their own O's, the appropriate shaded part and the motion or motions, and finally to obtain the resultant polygons (R). To be creative in this situation probably requires one to be "sensitive" to the fact that, (a) the shaded part need not be a polygon, since it can have arcs instead of straight line segments, (b) vertices of shaded part need not be the dots, (c) more than one shaded part can be selected at any one time, and (d) a combination of different motions (or glides) for the different shaded parts selected as a single response is permissible. "Conjecturing" may be more helpful in this case. A student can guess the resultant polygon first, and look for the appropriate shaded part(s) and motion(s). In so doing, a student is essentially "transforming" and "redefining" the problem to one that is similar to the Problem (1) of the pretest CG1, i.e. forming "appropriate" polygons or close geometrical figures, then looking for suitable shaded parts to produce these figures. One is more likely to produce more solutions through this transformation,

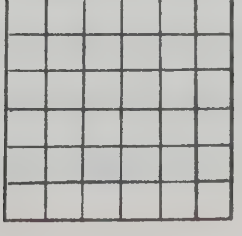
In short, we argue that the above two problem-situations

are quite relevant and suitable for students to utilize their knowledge in motion geometry to exhibit their mathematical creativity.

Creative Geometry Test II (CG2)

This test presents students with two general geometrical problem-situations that reflect the functioning of the three creative processes. Attempts were made, (a) to select items related to geometry concepts similar to that of the pretest CG1, (b) to design problem-situations closely resembling that of CG1, and (c) to provide situations where some of the concepts and skills learned in the 19 lessons on motion geometry could be utilized. The reasons for doing so were two-fold: (a) to enable us to assess the effectiveness of the two methods of instruction in terms of students' achievement on CG2, in the light of their initial performance on CG1, and (b) to provide optimal situations that facilitate creative thinking on the part of the students who had learned motion geometry through two different approaches, thus enabling us to attribute any differences of performance between the two groups to the effects of the treatments. We will now argue that the following two problems satisfy these requirements.

(1) You are given a SQUARE:

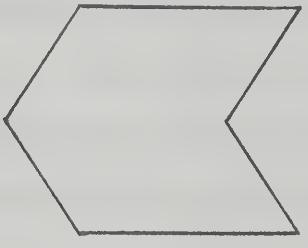


Cut the square into 2 parts of EQUAL AREAS.

You are asked to DRAW ALL THE POSSIBLE WAYS OF CUTTING by copying the square onto the given graph paper.

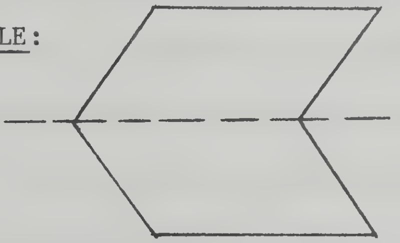
Draw as MANY CUTS as you can think of.

(2) This is a HEXAGON (a 6-sided polygon):



You are asked to SAY SOMETHING about this hexagon by DOING SOMETHING about it.

EXAMPLE:



The hexagon can be divided into 2 different quadrilaterals (4-sided polygons), in fact they are parallelograms.

Now go on to write as MANY IDEAS as you can think of about this hexagon.

DRAW A FIGURE FOR EACH STATEMENT YOU PUT DOWN.

The D of problem (1) consists of a square with 5 vertical and 5 horizontal lines of division. The R is the set of all possible pairs of geometrical closed plane figures, each pair comprising figures of equal area which make up the given square if put together. The O is the set of all lines that cut the square into two parts of equal area. Students are required to select their own O set to obtain the appropriate R set. To be sensitive in this situation necessarily implies the realization that (a) equality in area is not equivalent to congruence of figures, (b) cutting lines

can be non-straight segments, such as zigzag lines or curves, and (c) though the 10 internal dividing lines may be helpful for many symmetrical solutions, they can be conveniently discarded for many other solutions. Obviously, the ability to transform the problem into a more general situation of separating the square into two figures of equal area but not necessarily identical shapes entails a measure of mathematical creativity. Furthermore, there is the need to "guess" the various possible forms of such pairs of figures. Problem (1) presents therefore an open and divergent type of geometrical problem-situation.

Problem (2) affords a geometrical situation similar to that of CG1: Problem (1). D consists of the hexagon, O asks for appropriate mathematical operations, and R denotes the resulted mathematical statements. One possible operation was given, that is: "to separate the hexagon into two polygons". Students are prompted by the given example to be sensitive to the possible familiar shapes or properties of the resultant polygons. This problem can be redefined into many sub-problems, one of which is to draw line segments through the hexagon and observe the various resulting mathematical properties. A more creative transformation is to operate on the whole figure or the separated parts through the application of suitable motions to obtain new polygons. Some students may even hypothesize the possible end-result polygons before the actual transformations or motions. This problem-situation is therefore very flexible and provides much room for inventive divergent productions.

A closer examination immediately reveals the intimate resemblance between the test items of CG1 and CG2. They all focus on the forming of various polygons or closed geometrical plane figures, or the separation of a given geometrical figure. The second problems of both CG1 and CG2 are of the same format. Since CG2 was taken after the 6-week instruction on motion geometry, no explicit example was given for Problem (2) of CG2 to indicate the possible utilization of geometric transformations (motions) to convert several hexagons or the separated parts of a hexagon into new polygons. Nevertheless, concepts of geometric transformation are implicit in both problems of CG2. For problem (1), congruence pairs of figures can be obtained easily through symmetrical cuts, and the pairs of non-congruence figures can be efficiently and effectively verified for equality of area through the transformation of one figure into the other figure. In problem (2), applications of motions to the hexagon or its divided parts are good ways of achieving divergent and creative production of solutions. We therefore regard these two items as appropriate creative problem-situations for CG2.

Scoring Procedures for Creative Tests

The previous sections of this chapter have shown that the problems constructed for the three creative tests (CG1, CMG and CG2) are proper creative situations of school geometry calling upon the functioning of the three creative processes, sensitivity,

redefinition and conjecturing. We have also demonstrated that when students respond to these types of problem-situations, their inventive divergent productions (the response solutions) can be reasonably regarded as the manifestation of their mathematical creativity defined by the three processes (supra: 67). Furthermore, it was indicated that the functioning of these creative processes cannot be identified separately from the students' creative responses. Consequently, an operational definition of creativity has been postulated as "practical criteria" to facilitate the measurement of mathematical creativity (supra: 64).

Creative problem-solving in school mathematics is thus defined operationally in terms of the following characteristics exhibited in responses to creative problem-situations:

(1) Fluency of Ideas: the production of a large quantity of distinct mathematical solutions to a given problem.

(2) Diversity of Ideas: the production of a large quantity of distinct categories of concepts reflected in the mathematical solutions.

(3) Rarity of Ideas: the production of distinct and original categories of concepts reflected in the mathematical solutions.

For the purpose of measurement, these three characteristics of mathematical creativity are indicated respectively by three scores: (1) Fluency Score, (2) Diversity Score, and (3) Rarity Score. Each item of a creative test was scored on these three dimensions, and the respective sum for the two items formed the

three-variate (or triple) creative score for a student on that test. We now elaborate on the development and applications of this three-fold scoring procedure.

(1) Fluency Score: To assess the exhibition of "Fluency of Ideas" by a student in a mathematical problem-situation, a Fluency Score is assigned to the student's responses (solutions). The Fluency Score is a measure of the total number of mathematically appropriate and distinct responses produced by a student in a creative problem-situation.

Here, we should note a certain divergence in researchers' opinions on what is deemed "mathematically appropriate". Evans (1964: 49), for example, allows for situations where the student is unsure immediately of the correctness of her/his response. Under such circumstances, credit is accorded both mathematically correct and incorrect responses. On the other hand, Taylor-Pearce (1971: 53) and Boychuk (1974: 121) both reject mathematically incorrect or inappropriate responses. As Boychuk observes, the "elimination of incorrect and inappropriate responses resulted in a lower fluency score than would have occurred if every response had been accepted." Since this study is aimed at assessing the feasibility of enhancing mathematical creativity in junior high students through creative instruction, it was decided to follow the latter's viewpoint. Thus correct and relevant responses are limited to the mathematical domain, and more conservative fluency scores adopted.

(2) Diversity Score: To assess the exhibition of "Diversity of Ideas" by a student in a creative problem-situation, a

Diversity Score is assigned to the student's responses. The Diversity Score is a measure of the total number of distinct categories of concepts embedded in the mathematically appropriate and distinct responses produced by a student in a creative problem-situation.

The Diversity Score of the present study is an improvement of Taylor-Pearce's (1971) "Variety Score" which in turn is a modification of Guilford and Torrance's "Flexibility Score". Taylor-Pearce (1971: 63-78) established deductively that every "mathematically appropriate" response implies a finite number of concept-sets.[3] All the concept-sets underlying a student's total responses to a problem-situation can be logically reduced to a "minimal number" of distinct and exclusive sets. This number then constitutes the student's "variety score" for that problem.

There is a limitation, however, to this technique. The mathematical criteria that determine the basic concept-sets involved in each response depend upon the judgement of the test scorer (Taylor-Pearce, 1971: 71). In addition, it is difficult to justify the connection between the psychological processes of mathematical creativity and the overt responses using criteria which consist of purely mathematical components. For these reasons, a new scoring procedure will now be proposed which will (a) take into consideration both psychological factors and mathematical concepts, and (b) reduce the subjectivity of scoring and facilitate replication studies.

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[3]The "finite-ness" of concept-sets was not stated explicitly by Taylor-Pearce. However, this "condition" was implied in his arguments and assumed for his mathematical analysis.

For every problem of the creative tests, a logical analysis is first undertaken to determine the major dimensions involved in solving the problem. The possible scope of the D (domain), O (operation) and R (range) is anticipated by considering the likely mathematical background of the target population. In addition, the conceptual definition of creativity should be taken account of in defining the relevant psychological dimensions. Each of these mathematical and psychological dimensions are then further analysed and broken down into a finite number of significant Diversity Categories to incorporate both relevant mathematical concepts and creative processes defined for our study. These diversity categories are established independently of students' responses. However, the special feature of the total sample of students' responses, if any, may well prompt a minor revision of the categories, to incorporate, for example, unanticipated creative dimensions or categories, or to delete empty dimensions or categories from our scoring scheme. A set of Diversity Categories is thus established for each creative problem to serve as scoring criteria for assigning the Diversity Score to students' responses to that problem. For a creative problem, the total number of these "diversity categories" embedded in the responses of a student is her/his "diversity score" for that item. Each category is counted only once for each creative problem.

Since the Rarity Score is also based upon the same "diversity categories", an illustration of the new scoring procedures for both diversity score and rarity score will be given

in section (4).

(3) Rarity Score: To assess the exhibition of "Rarity of Ideas" by a student in a creative mathematical situation, a Rarity Score is assigned to the student's responses. The Rarity Score is a measure of the infrequency or uncommon-ness of the distinct diversity categories embedded in the mathematically appropriate and distinct responses produced by a student in a creative problem-situation.

The Rarity Score of our study is also an improvement of Taylor-Pearce's (1971: 76-78) "Novelty Score" which in turn is a modification of Guilford and Torrance's "Originality Score". The underlying assumption of our Rarity Score, quite explicit in our conceptual definition of mathematical creativity, is clearly that a more frequently given category of concepts (diversity category) is not as original and creative as a less frequent one. The Rarity Score of a diversity category exhibited in the responses for a creative problem is either 4, 3, 2, 1, or 0, according to its statistical frequency among the sample of students involved in our study. For each problem, each distinct "diversity category" is counted only once for a student. The Rarity Score on a problem is the sum total of the scores for each diversity category. The criteria for awarding scores are as follows (Evans, 1964: 51; Boychuk, 1974: 124). For ease of scoring, the corresponding frequency is computed for our sample of 41 students:

Diversity Category by % of Students	Corresponding Frequency	Rarity Score
0 - 20	1 - 8	4
21 - 40	9 - 16	3
41 - 60	17 - 24	2
61 - 80	25 - 32	1
81 - 100	33 - 41	0

A brief note on the approaches of other researchers is relevant here. In many studies, scoring for novelty or originality is based on uncategorized students' responses (Dunn, 1975: 329). Rather than ranking the "categories of concepts" according to their "infrequency", such studies rank the responses and assign novelty scores to each response. This approach suffers from two weaknesses.

First, there is the possibility of obtaining a great variety of responses with very low frequency, which renders their novelty score meaningless. Secondly, there inevitably arises the problem of the "acceptability of responses" (Taylor-Pearce, 1971: 77) or the "relevance of response" (Treffinger, Renzulli and Feldhusen, 1971: 111). Since trivially adequate responses tend to have lower frequency, applying criteria of statistical infrequency directly to students' responses will give high novelty scores to such uncommon responses. This in turn, penalizes the more discriminating and sensitive students who do not bother to write down "insignificant" answers. An attempt to resolve this difficulty is Taylor-Pearce's (1971: 78) use of the statistical frequency to establish the upper bound of a response. If this response is also implied in another

response, then the novelty score for the first response should not be higher than the second one. The major problem here clearly lies where the trivial uncommon response is the only single response a student gives to a particular problem, in which case the proposed technique is useless.

On the other hand, the above difficulties will not arise if we assign a Rarity Score (or novelty, or originality score) not to "uncategorized" responses, but to the significant "diversity categories" identified for our Diversity Scores. This is because such "categories" have been selected on the basis of their relevancy and significance in either the mathematical content domain or the domain of creative problem solving processes. Since a trivial response to a mathematical situation under our criteria is bound to consist of routine and obvious categories of mathematical concepts or psychological processes, fairly low rarity scores will be properly assigned to these categories.

(4) Illustration of the Scoring Procedures: Through a critical evaluation and incorporation of the scoring techniques employed by previous creativity research in school mathematics, we have developed a more rigorous and objective procedure for assessing the fluency, diversity and rarity of students' creative responses. As Treffinger and Poggio (1972: 266) concluded in their assessment of the complexity and intricacy involved in scoring "open-ended" measures of creative thinking,

> Problems related to test scoring are very important in the measurement of creativity. In addition to research on the comparability of scores derived from different tasks and different methods of testing, studies should also be conducted which investigate new methods and criteria for scoring.

The development of Diversity Categories for diversity and rarity scores in this study represents, in effect, an attempt to assess students' creative responses in mathematical problem-situations more objectively and accurately. To illustrate the application and usefulness of our procedures, we will discuss in detail Problem (1) of posttest CG2.

This problem (*supra*: 143) is a modified version of Boychuk's (1974: 58) Sensitivity I (The Square). In this situation, a square with 5 vertical and 5 horizontal lines of division is given, and students are required to cut the square into "Two Parts of Equal Areas". We have argued that this is an appropriate creative problem-situation reflecting the creative processes of sensitivity, redefinition and conjecturing. It appears to us that creative and divergent solutions to this problem are likely to result from variations on four dimensions: (I) the nature of the cut line drawn, (II) the orientation of the cut line, (III) the shapes of the resultant two parts, and (IV) the use of the irrelevant information of the 10 internal dividing lines. These four aspects take into consideration the mathematical concepts of equal area, congruence and closed geometrical figures, as well as the creative abilities to "see" the possibility of equal areas for non-congruent figures, and to discard irrelevant information. Consequently, a set of

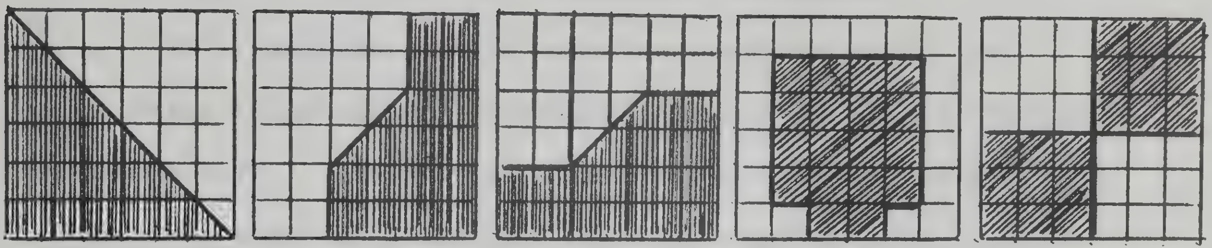
17 "Diversity Categories" can be developed upon these four major dimensions. Table 6.1 shows the Diversity Categories with their frequency and respective Rarity Scores.

TABLE 6.1

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE GEOMETRY TEST II (CG2): PROBLEM (1)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Nature of the Cut Line:		
a) Simple straight line.	41	0
b) Simple regular zig-zag line.	37	0
c) Irregular zig-zag line.	16	3
d) Simple curve.	1	4
e) Combination of straight line segments and curve, or complex curve.	1	4
II. Orientation of the Cut Line:		
a) Horizontal/Vertical.	41	0
b) Diagonal.	41	0
c) Horizontal and vertical.	31	1
d) Horizontal and diagonal.	7	4
e) Vertical and diagonal.	12	3
f) Horizontal, vertical and diagonal.	14	3
III. Shapes of the Parts Resulted:		
a) Two congruent parts.	41	0
b) Two incongruent parts.	16	3
c) Use of two or more than two pieces to make a part.	7	4
IV. Use of Irrelevant Information:		
a) Cut along the given straight line(s) and points of intersection.	41	0
b) Cut along the given line(s) or point(s), but not both.	41	0
c) Cut not along both given line(s) and point(s).	8	4

Consider the following sample of responses of a student:



Since the second and third responses are basically the same, the fluency score for this student is 4. According to Table 6.1, the diversity categories involved and their respective rarity scores are as follows:

Diversity Category		Rarity Score
(I):	a	0
	b	0
(II):	b	0
	c	1
	d	4
	e	3
(III):	a	0
	b	3
	c	4
(IV):	a	0
	b	0

Diversity Score = 11   Rarity Score = 15

This example demonstrates the two advantages in using these pre-established "diversity categories" to measure diversity and rarity of ideas. First, since these criteria-categories are developed independently of any set of data, they are not sample-biased.[4] Secondly, they provide a logical basis for discussion with regard to

[4]These very same 17 diversity categories were initially developed for scoring a similar item used in our pilot study, where 100 students were involved.

the validity of the test items and scoring procedures.

It is relevant at this point to briefly argue for the superiority of our scoring procedures over that employed by Boychuk (1974: 132-135). For this sample sensitivity problem, Boychuk first grouped the initial 50 different responses from the whole sample into 23 categories. Later, the number of categories was reduced to 13 in order to obtain adequate and meaningful novelty scores, which is tied logically to her variety score. These 13 categories then served as criteria for assigning variety scores and novelty scores to responses. The major deficiency in this categorization to give variety scores lies in the lack of explicit mathematical or psychological criteria used to distinguish between the different categories. Complexity, symmetry, congruence, and configural difference were apparently taken into account but no clearly spelled out criteria and rationale given. Such lack of objective and explicit criteria, as well as the sample-biased nature of the scoring procedures, will undoubtedly limit the feasibility of replicating Boychuk's study.

Consequently, following a similar trend of arguments as that of CG2: Problem (1), all creative problems of our tests were scored according to the respective sets of "Diversity Categories" developed.

(5) Diversity Categories and Rarity Scores for Creative Problems: Earlier in this chapter (*supra*:137f), we performed a logical analysis on mathematical concepts and creative processes involved in each of the creative problems constructed for the

three creative tests, CG1, CMG and CG2. Based on such analyses and the type of considerations illustrated in the previous section, sets of Diversity Categories with the respective Rarity Scores were thus developed for the scoring of the three creative tests: CG1, (Tables 6.2 and 6.3), CMG (Tables 6.4 and 6.5), and CG2 (Tables 6.1 and 6.6).

TABLE 6.2

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE GEOMETRY TEST I (CG1): PROBLEM (1)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Nature of the Sides:		
a) Straight lines.	41	0
b) Curve.	1	4
c) Combination of straight lines and curves.	2	4
II. Topological Properties:		
a) Convex figure.	41	0
b) Concave figure.	41	0
c) Figure with all vertices of degree 2.	41	0
d) Figure contains vertices of degree greater than 2.	27	1
III. Number of Sides/Arcs for a Figure:		
a) 1 (a loop)	1	4
b) 2 (2 arcs/sides)	2	4
c) 3	40	0
d) 4	41	0
e) 5	39	0
f) 6	41	0
g) 7	39	0
h) 8	10	3
i)10	1	4

TABLE 6.3

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE GEOMETRY TEST I (CG1): PROBLEM (2)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Separating the Rectangle:		
1) Nature of dividing lines:		
a) Straight line(s).	41	0
b) Curve.	8	4
c) Zig-zag lines.	5	4
2) Types of figures resulted:		
a) Triangle.	41	0
b) Square.	37	0
c) Rectangle.	33	0
d) Parallelogram.	2	4
e) Trapezoid.	12	3
f) Other Polygons.	12	3
g) Circular figure, or figure contains arc(s).	1	4
3) Number of different types of figures formed in a rectangle:		
a) 2	41	0
b) 3	23	2
c) 4	7	4
d) More than 4	1	4
4) Congruence of parts:		
a) Cut results in congruent figures in a rectangle.	40	0
b) Cut results in some incongruent figure(s).	31	1
5) Operation on separated parts to form new figure(s):		
a) For each cut, only one new figure is formed.	2	4
b) For each cut, more than one figure is formed.	1	4
II. Operation with Whole Rectangle to form Larger Figure:		
1) Number of rectangles used:		
a) 2	15	3
b) 3	4	4
c) 4	5	4
d) 6	2	4

II. 2) Number of different figures formed for given number of rectangles used:		
a) 1	17	2
b) 2	1	4
3) Additional operations performed, e.g. separating the new figure formed.	10	3

TABLE 6.4

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE MOTION GEOMETRY TEST (CMG): PROBLEM (1)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Motion(s) Performed:		
1) Slide:		
a) (0,0)	1	4
b) (1R,0)	35	0
c) (0,1D)	32	1
d) (1R,1D)	32	1
2) Reflection (Nature of the Mirror Line):		
a) Using BC.	37	0
b) Using AC.	40	0
c) Using AB.	32	1
d) $\perp$ to AB.	15	3
e) // to AC, $\neq$ AC.	12	3
f) // to BC, $\neq$ BC.	7	4
g) // to AB, $\neq$ AB.	32	1
h) Others.	5	4
3) Turn:		
a) ½-turn.	32	1
b) ¼ (or 3/4) turn.	34	0
c) Revolution.	4	4
d) Others.	2	4
e) Center on dot.	41	0
f) Center Not on dot.	16	3
4) Combination:		
a) Slide, Reflection.	22	2
b) Reflection, Slide ($\neq$ Slide-reflection).	2	4
c) Slide, Slide.	3	4
d) Reflection, Reflection.	3	4
e) Turn, Reflection.	4	4
f) Slide, Turn.	4	4

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. 5) Number of equivalent motions:		
a) 1	30	1
b) 2	2	4
c) 3	1	4
II. Images:		
1) All vertices on dots.	41	0
2) Some vertices Not on dot(s).	12	3

TABLE 6.5

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE MOTION GEOMETRY TEST (CMG): PROBLEM (2)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Shaded Part Selected:		
1) Number of such parts:		
a) 1	41	0
b) 2	7	4
c) 3	3	4
d) 4	6	4
e) Incongruence parts.	4	4
2) Information related to dots:		
a) All vertices of shaded part or image on dots.	41	0
b) Some vertices of shaded part not on dots.	5	4
c) Some vertices of image not on dots.	4	4
3) Shape of shaded part:		
a) Straight line segment.	1	4
b) Square.	35	0
c) Triangle.	40	0
d) Rectangle.	31	1
e) Other polygon.	23	2
f) Circular figure.	1	4
g) Figure with arc and straight segment.	1	4
4) Topological properties:		
a) Convex.	39	0
b) Concave (besides the given example figure).	37	0

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
II. Motion Performed:		
1) Slide.	37	0
2) Reflection.	37	0
3) Turn.	23	2
4) Slide-reflection.	4	4
5) For more than one shaded part selected; single motion used.	5	4
6) For more than one shaded part selected, more than one motion is used.	5	4
7) Shaded part and image overlap.	1	4

TABLE 6.6

DIVERSITY CATEGORIES AND RARITY SCORES FOR CREATIVE GEOMETRY TEST II (CG2): PROBLEM (2)

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
I. Cutting the Hexagon:		
1) Nature of cut line:		
a) Cut line joining some points.	41	0
b) Cut line joining no point.	26	1
c) Straight line(s).	41	0
d) Simple zig-zag line(s).	7	4
e) Complex zig-zag line(s).	2	4
2) Types of polygon formed:		
a) Triangle.	39	0
b) Rectangle.	11	3
c) Square.	2	4
d) Parallelogram.	14	3
e) Trapezoid.	10	3
f) Quadrilateral.	29	2
g) Polygons with more than four sides.	37	0
3) Number of different types of polygons formed in each hexagon:		
a) 1	26	1
b) 2	40	0
c) 3	11	3
II. Motion Performed:		
1) Motion on the Hexagon.	4	4
2) Motion on Part(s) of Hexagon.	11	3
3) Motion: Slide.	7	4
4) Motion: Reflection.	8	4
5) Motion: Turn.	5	4
6) Resulted in obvious figure(s).	7	4
7) Resulted in ingenious figure(s).	1	4

DIVERSITY CATEGORY	FRE-QUENCY	RARITY SCORE
III. Properties Observed:		
1) Angle properties.	11	3
2) Congruence of side(s).	4	4
3) Number of diagonal(s).	1	4
4) Congruence of whole polygon(s) resulted.	11	3
5) Parallelism.	4	4

Validity and Reliability of Creative Tests

Unlike the traditional standardized intelligence or achievement tests, the creative tests employed to assess mathematical creativity in our study were specially devised and applied in the course of the investigation. The value and significance of our conclusions depend heavily upon the validity of our measuring instruments and the reliability of the data obtained through these instruments. As Aiken (1973: 32) points out, inadequate measuring instruments severely limit the value of research findings, and studies in mathematical creativity suffer from "unreliable or invalid measuring instruments." In this section, attempts will be made to validate our creative tests and estimate their reliability in the present study.

(1) Validity

Validity asks the question: "Are we measuring what we think we are measuring? The emphasis in this question is on What is being measured" (Kerlinger, 1973a: 457). Different answers can be provided, depending upon the many different applications of the instruments. Thus, Cronbach (1971: 447) asserts that "one validates, not a test, but an interpretation of data arising from a specified

procedure. . . . Since each application is based on a different interpretation, the evidence that justifies one application may have little relevance to the next." Three types of validity are hence distinguished for varied types and applications of tests: (a) content validity, (b) criterion-related validity, and (c) construct validity (American Psychological Association, 1973).

A test generally has two major uses: (a) for making decisions or predictions about the student tested, and (b) for describing her/him. Decisions are intended to anticipate the student's later performance on some related "criterion", and such a predictive use of a test is thus dependent upon criterion-related validity. Descriptive use, however, aims at interpreting differences between students' performance on a test, therefore relies on content validity or construct validity (Cronbach, 1971: 445; Mehrens and Lehmann, 1973: 124). Since the present study seeks not to make decisions for student placement or predict students' future performance, but to describe and interpret students' performance before and after two different treatments, it is sufficient to consider the content and construct validity of the three specially designed creative tests.

(a) Content Validation: Kerlinger (1973a: 457) has stated categorically that "a test or scale is valid for the scientific or practical purpose of its user." With respect to the many validities, Mehrens and Lehmann (1973: 135) also argue that "content validity of achievement tests is by far the most important type of validity", and construct validity is necessary only when "one wishes to use test data as evidence to support or refute a psychological theory."

A similar viewpoint has been expressed by Seibel (1968: 288). Hence, the validity of our creative tests is best thought of as the extent to which the content or substance of these measures are "representatives of the content or universe of content of the property being measured" (Kerlinger, 1973a: 458). As we have shown in our discussion of the related mathematical content and creative problem solving processes embedded in the 6 items of the 3 creative tests (CG1, CMG and CG2), the test items constructed do in fact reflect the basic components of the universe of creative behaviors defined for our study (*supra*:137f). Content validity of these three tests can therefore be satisfactorily assumed.

(b) *Construct Validation*: Construct validity is "the degree to which certain theoretical or explanatory constructs can account for item responses and test performances" (Payne, 1974: 258). In this respect, construct validation calls for evidence to enable us to conclude with confidence that the three creative tests really measure the three creative problem solving processes (constructs) defined in our conceptual model. It would, in fact, amount to the validation of our theoretical model of mathematical creativity upon which the three tests were constructed (Cronbach, 1970: 143; Kerlinger, 1973a: 461). This theoretical model has already been shown to be a valid model by Boychuk (1974) through a factor analytical procedure, the *Image Analysis*. Nevertheless, a brief discussion will be beneficial in providing a better ground for any conclusions and implications drawn from the outcomes of our inventive treatment.

Our conceptual model is derived essentially from Guilford's structure-of-intellect theory (1967) adapted to the discipline of mathematics. Guilford's model as well as many other similar models, attempts to establish the existence of intellectual factors (constructs) that are consistent with the postulated relations among the constructs to be defined (Merrifield, 1974: 416). Mulaik (1972: 1-10) shows that such a structural theory or model, which regards phenomenon of human behaviors as an aggregate of elemental components (factors or constructs) interrelated in a lawful way, can be adequately validated through factor analytical approaches. He further demonstrates that the technique of Image Analysis allows us to "formulate a better model of factor analysis as well as to unify concepts of factor analysis with those in the area of measurement in the behavioral sciences" (Mulaik, 1972: 187). We will therefore perform separately an image analysis on the scores of each creative test. Since each of the two items of a creative test was scored on three dimensions (fluency, diversity and rarity), the six scores thus obtained can be regarded as six interrelated measures of the three underlying creative processes (the constructs): sensitivity, redefinition and conjecturing. Construct validity is consequently tenable if three interpretable factors could be extracted for each creative test.

It is a general practice in factor-analysis literature for a factor loading of .33 to be the minimum absolute value to be interpreted since approximately 10% of a variable's variance is accounted for by this minimum loading (Willemsen, 1974: 151).

However, since our measuring instruments were not constructed for theory validation, we will follow Boychuk (1974: 160) in selecting the more liberal value of .30 as the criterion-loading value indicating the existence of factors needing interpretation. Such factors under Varimax Rotation are given in Tables 6.7, 6.8 and 6.9, with all loadings having absolute values greater or equal to .20 on these extracted factors.

TABLE 6.7

SUMMARY OF IMAGE-ANALYSIS UNDER VARIMAX ROTATION FOR THE SIX SCORES OF CG1

VARIABLE	FACTOR				COMMUNALITIES
	I	II	III	IV	
PROBLEM (1):					
Fluency	.27	.31	.45	--	.38
Diversity	--	.92	--	--	.84
Rarity	--	.84	.26	--	.81
PROBLEM (2):					
Fluency	.78	--	--	--	.64
Diversity	.81	--	.23	.34	.84
Rarity	.86	--	--	--	.77
EIGENVALUES	2.29	1.77	.15	.10	
% of Total Variance	38.2%	29.5%	2.5%	1.7%	

Sum of Communalities = 4.28

Total Variance Accounted for = 71.3%

TABLE 6.8

SUMMARY OF IMAGE-ANALYSIS UNDER VARIMAX ROTATION FOR THE SIX SCORES OF CMG

VARIABLE	FACTOR I	FACTOR II	FACTOR III	COMMUNALITIES
PROBLEM (1):				
Fluency	.85	.30	--	.81
Diversity	.84	.32	--	.82
Rarity	.82	.34	--	.80
PROBLEM (2):				
Fluency	.46	.62	.30	.69
Diversity	.36	.87	--	.89
Rarity	.27	.89	--	.87
Eigenvalues	2.52	2.24	.11	
% of Total Variance	42.0%	37.5%	1.8%	
Sum of Communalities = 4.88				
Total Variance Accounted for = 81.3%				

TABLE 6.9

SUMMARY OF IMAGE-ANALYSIS UNDER VARIMAX ROTATION FOR THE SIX SCORES OF CG2

VARIABLE	FACTOR I	FACTOR II	FACTOR III	COMMUNALITIES
PROBLEM (1):				
Fluency	.60	.44	--	.56
Diversity	.95	--	--	.91
Rarity	.94	--	--	.91
PROBLEM (2):				
Fluency	--	.41	.62	.57
Diversity	--	.95	.23	.97
Rarity	--	.93	.30	.98
Eigenvalues	2.21	2.14	.55	
% of Total Variance	36.8%	35.6%	9.2%	
Sum of Communalities = 4.90				
Total Variance Accounted for = 81.7%				

The four factors extracted for CG1 (Table 6.7) account for 71.3% of the total variance. Factor I appears to be the conjecturing process, indicated by the high loadings for Problem (2) which requires students to search for their own O (operation) and R (range). Factor II may well be the process of sensitivity, since Problem (1) requires awareness of possibilities. Factor III could be redefinition, while the unknown Factor IV only explains less than 2% of the total variance.

The three factors extracted for CMG (Table 6.8) account for 81.3% of the total variance. Factors I and II clearly denote the processes of sensitivity and conjecturing, whereas the third factor could be redefinition. The minor contribution of redefinition to students' performance is probably due to the fact that without transforming the two problems of CMG, students could still produce very many appropriate solutions.

The three factors extracted for CG2 (Table 6.9) account for 81.7% of the total variance. Since both problems loaded on Factor II, especially Problem (2) which is similar to that of CG1: Problem (2), it indicates the process of conjecturing. Factor I is undoubtedly the process of sensitivity which is essential for the solutions of Problem (1). Factor III could be redefinition, because of the implied transformation of Problem (2) (<u>supra</u>: 145).

The set of factors extracted for each of the three creative tests explained a rather significant portion of the total variance. The image analysis has thus established the needed construct validity of our three creative tests.

(2) Reliability

Reliability asks the question: "Are the measures obtained from a measuring instrument the 'true' measures of the property measured?" (Kerlinger, 1973a: 443). Affirmative answers essentially imply that the instrument did produce consistent measures for the same thing (Mehrens and Lehmann, 1973: 102). Maguire and Hazlett (1969: 125) have argued strongly that "regardless of the kind of data, the question of reliability of a measurement is fundamentally a question of the consistency of the measurement." Furthermore, they stated that of the many ways for calculating indices of consistency, "the one that is used should be determined by the use to which the measurement is to be put." The primary objective of the constructed creative tests was to detect differences of students' capacity in solving mathematical problems creatively. For each of the two-item creative tests, the question basic to reliability is: "Are the scores on the two items of the test consistent?" The answer is "Yes" if students scoring higher than others on one item tend to score high on the other too. Statistically, this implies that students' scores on the two items of a creative test are correlated significantly. We need therefore to test the null hypothesis of independence of students' responses to the two items.

The independence of two variables with a joint bivariate normal distribution can be tested by "simple correlation", while the independence of one variable with the other p-1 in a multi-normal system can be tested through "multiple correlation". Since the two items were scored on three dimensions (fluency, diversity

and rarity), each consists of three variables. In this case, we are therefore concerned with a system of (p+q) multinormal variables, where the independence between a set of p variables and another set of q variables, has to be verified or falsified. The generalized multiple correlation, i.e. the <u>Canonical Correlation</u> model is needed for such purposes (Anderson, 1966: 166; Timm, 1975: 348; Morrison, 1976: 254).

If Σ_{12} denotes the covariance matrix among the p and q variates (or variables), then $\Sigma_{12} = 0$ is a necessary and sufficient condition for the independence of the two sets of variates. The null hypothesis is hence (Morrison, 1976: 254):

$$H_0: \Sigma_{12} = 0, \text{ against } H_1: \Sigma_{12} \neq 0.$$

Table 6.10 gives the results of the tests for independence of the two sets of variates (i.e. the fluency, diversity and rarity scores of the two items) for the 3 creative tests: CG1, CMG and CG2.

TABLE 6.10

TEST OF INDEPENDENCE USING CANONICAL CORRELATION FOR CG1, CMG, AND CG2 (9 and 85.3 d.f.'s)

TEST	F	P	At .05 level, H_0 is :
CG1	1.51	.157	Not Rejected.
CMG	3.37	.001	Rejected.
CG2	3.51	.001	Rejected.

The significance tests at the .05 level indicate that the H_0 is rejected for CMG and CG2. It is concluded therefore that the two sets of fluency, diversity and rarity scores of the two items of each of these two creative tests are dependent. In other words, measures on the two items of CMG and CG2 are interrelated in some significant way, and we can confidently say that these two tests are reliable because each of them had produced a set of consistent measures.

For the pretest CG1, we cannot reject the null hypothesis at the .05 level. The reliability of this instrument is thus dubious. As indicated by our image analysis in the previous section, this is probably due to the fact that CG1 also involves an unknown factor other than the three creative processes. Since CG1 was administered prior to the introduction of the treatments, and it was totally novel to students' past experiences in mathematical problem-situations, we may reasonably suspect that the fourth factor falls into the area of motivation and attitude.

Alternatively, if we regard the 6 scores on the two items of each creative test as scores on 6 sub-items contained in the entire test, a Cronbach Alpha (α) reliability coefficient can be computed for each test. The alpha reliabilities of CG1, CMG and CG2 are respectively .61, .85, and .72. According to Seibel's (1968: 272) criterion, CMG and CG2 are therefore adequate for purposes of group comparison, while CG1 is not satisfactorily reliable. Consequently, CMG and CG2 can be regarded as valid and reliable measuring instruments. The results of this study where CG1 is involved

however, should be treated with some caution.

THE TRADITIONAL TESTS

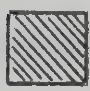
Besides the three creative tests, 6 traditional measures were also obtained for the present study: GR7, A5T, MAR, SEC, TMG and TAR. The validity and reliability of these measures are given below.

Validity of Traditional Tests

(1) Transfer Area Test (TAR): Since the topic immediately prior to the two treatments was on metric area units with simple calculation of areas of square and rectangle, TAR was designed to assess students' ability to apply concepts and skills learned in motion geometry to area-finding problem-situations. The test consists of 14 items focussing on a square, a rectangle, 2 triangles, a parallelogram, and 9 other non-standard closed geometrical plane figures. A point is awarded for (a) a correct solution, (b) a correct application of computational procedure, and (c) a correct application of geometric transformations. Hence, a student can score a maximum of 2 points on each of the first two items where the criterion (c) is not applicable, and a maximum of 3 points for each of the 12 remaining items. The complete set of 14 items are given in Appendix H. These test items only require the simple concept of "Area" in geometry. To avoid ambiguity and confusion, the following instructions and examples were given:

In order to measure AREA, we use a UNIT of area which is a SQUARE of side 1 unit. For this set of exercises, our unit is 1 $cm.^2$

1 cm.

1 cm. 

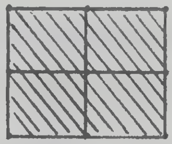
Find the area of each of the shaded regions.

Show all the steps of your calculation.

Two worked examples are given as hints:

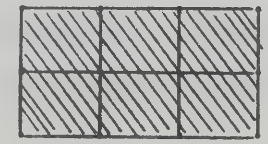
AREA = 1 x 1 $cm.^2$
= 1 $cm.^2$

(a)



Area = 2 x 2 $cm.^2$
= 4 $cm.^2$

(b)



Area = 2 x 3 $cm.^2$
= 6 $cm.^2$

Since students had learned to find only the area of a square or a rectangle, it would be necessary for them to search for some non-standard methods in order to compute the areas of the given triangle, parallelogram and other novel geometrical figures. We would expect them to utilize some of the motions (glides) to transform these other 12 figures into squares or rectangles and hence solve the problems. TAR therefore provides geometrical problem solving situations suitable for the utilization of knowledge of motion geometry learned in the 19 lessons.

The traditional achievement tests such as GR7, A5T, and MAR not only reflect students' performance on certain mathematical topics, but also indicate to some extent the students' ability in tackling mathematical problem-situations. Consequently, these tests should correlate among themselves, as well as with TAR. Table 6.11 shows that GR7, A5T and MAR correlate significantly with one another and with TAR, (r = .53, .49 and .35 respectively). Furthermore, since SEC indicates students' Euclidean level of geometric maturity,

which reflects students' ability to perform geometric transformation in Euclidean 3-space, we would expect TAR to correlate significantly with SEC. This prediction was confirmed by the correlation coefficient of .61. Being a test that can be solved easily through the application of geometric transformation, TAR should also correlate with TMG, the Traditional Motion Geometry Test. The computed r of .56 confirmed our expectation. These results thus strengthen the construct validity of TAR.

TABLE 6.11

CORRELATION MATRIX FOR 6 TRADITIONAL TESTS*

TEST	1	2	3	4	5
1. GR7 (Grade 7 Math.)					
2. A5T (Average of 5 Tests)	.92				
3. MAR (Metric Area)	.52	.52			
4. SEC (Piagetian Sectioning)	.55	.46	.33		
5. TMG (Trad. Motion Geometry)	.77	.75	.51	.59	
6. TAR (Transfer Area)	.53	.49	.35	.61	.56

*All correlations significant at the .05 level.

(2) Piagetian Sectioning Test (SEC): A test of students' performance on the topic of sectioning solids was first designed by Boe (1966) in conformity with Piaget and Inhelder's (1967), and subsequently modified and used by others (Davis, 1973; Bober, 1973; Pothier, 1975). Though some minor modifications were made to improve

some details of this test (SEC), the content and format of SEC used in this study remain essentially unchanged from previous usage. All the above cited studies have demonstrated the validity of SEC beyond reasonable doubt.

(3) Traditional Motion Geometry Test (TMG): The 40 items of TMG were part of the 66-item two-part major evaluation test designed by the Edmonton Separate School Board to assess the effectiveness and efficiency of the newly introduced junior high mathematics program. The validity of TMG can be assumed on the basis of the all embracing nature of its content material covered, as well as the expertise of the school board. These 40 items are given in Appendix E.

(4) Pretests: GR7, A5T and MAR: The first two are students' grade point averages on mathematics achievement tests, and the third is an achievement test score on a particular topic -- metric area units and simple area computation. Essentially, these measures contain traditional content-oriented type of test items. Their content validity can be assumed, since most of these items were based on sample test materials or instructional materials designed by the school board. The correlations among these tests indicated in Table 6.11 are all significant at the .05 level, which fact contributes to the construct validity of GR7, A5T and MAR.

Reliability of Traditional Tests

Since GR7, A5T and MAR were obtained from students' records, and information on individual test items were not available, no reliability could be computed for these 3 tests. Table 6.12 gives the types and coefficients of reliability for SEC, TMG and TAR.

TABLE 6.12

TYPES AND COEFFICIENTS OF RELIABILITY FOR SEC, TMG AND TAR

TEST	NUMBER OF ITEMS	TYPE	COEFFICIENT
SEC (Piagetian Sectioning)	32	KR-20	.87
TMG (Trad. Motion Geometry)	40	KR-21	.86
TAR (Transfer Area)	14	ALPHA	.90

Kuder-Richardson formula (KR-21) was employed for TMG because it seems reasonable from their content that all the 40 items are of equal difficulty. If this assumption is really not tenable, then .86 is in fact an underestimation of the true reliability of TMG (Mehrens and Lehmann, 1973: 113). Cronbach's Coefficient Alpha (α) was used for TAR because it is a generalization of KR-20 formula when the items are not scored dichotomously, as in the case of SEC. As Seibel (1968: 272) states, "reliabilities in the 70's or low 80's are adequate for most purposes that involve using summaries of test scores as information about groups". In effect, the high reliability coefficients of SEC, TMG and TAR are certainly more than adequate for

group comparison.

THE OBSERVER RATING SCALE

Validity

The _Observer Rating Scale of Teacher Behavior_ employed in the present study was developed by Naciuk (1968) to differentiate teachers' behaviors in "_mathematizing mode_" from that of the "_expository method_" of instruction in mathematics classrooms. We have already discussed at length that the "mathematizing mode" of teaching is a sequence of inventive type of mathematics instructions that aims at encouraging divergent thinking in students. Furthermore, the "expository method" defined by Naciuk (1968: 4) is very similar to the direct expository approach defined in our study for the TM as well as the DC phase of the IM. Hence this _Rating Scale_ can justifiably be used to differentiate the "_inventive component_" from the "_traditional component_" between the IM and the TM, as well as within the methods.

The _Rating Scale_ contains 6 dimensions: (a) teacher omniscience, (b) introduction of generalization, (c) control of pupil interaction, (d) method of answering questions, (e) use of student responses, and (f) method of eliminating false concepts. Five items were designed for each dimension. Each item was rated on a five-point scale, under the assumption that the two methods (IM and TM) could be regarded as extremes on a bi-polar continuum where a rating of 2 or below indicates a "traditional" component,

and above 2 an "inventive" component in classroom instruction. The 30 items (see Appendix B) appear to describe quite appropriately the display function (dimensions a, b, d and f) and the control function (dimensions b, c, d, e and f) of instruction. The content validity of the instrument is thus tenable. The construct validity is guaranteed by the successful application in Naciuk's (1968) studies in evaluating the potential of a "method in-service education program" for effecting change in the teaching methods of 7 teachers of Mathematics 20.

Reliability

Twelve classroom observations were obtained for each method, 6 on the Development phase and 6 on the Discussion phase of instruction. For each observation, the teacher's behavior was rated on all the 30 items. The sum total of the 5 items under a dimension constituted the rating on that dimension. Six ratings were thus obtained for each observation (visit). Since we have argued earlier that reliability is fundamentally a question of consistency of a measuring instrument, it follows that the rating scale is reliable if it rated the teacher's classroom behavior consistently on the 6 dimensions of each of the 24 observations. Technically, we could regard each observation as a subject tested and the 6 ratings as the subject's scores on 6 items of the test -- the Rating Scale. Hence the Cronbach Coefficient Alpha (α) of the rating scale is a good index of the required reliability.

Since four different comparisons were performed on teacher's behavior in the two instructional methods, 4 separate reliability coefficients were computed. Table 6.13 indicates the Alpha reliabilities for these four cases. For comparisons between the respective instructional phases of the IM and TM, and between the two phases of the IM, reliabilities are respectively .89, .92, and .93 -- all of which are more than adequate for our purposes (Seibel, 1968: 272). The reliability of .67 for the comparison between the two phases of the TM, though it did not meet Seibel's criterion of "in the 70's", was sufficiently close for our purposes.

TABLE 6.13

ALPHA RELIABILITIES FOR THE OBSERVER RATING SCALE OF TEACHER BEHAVIOR (No. of Items = 6)

COMPARISON	ALPHA
IM vs. TM : Development of Concepts	.89
IM vs. TM : Discussion of Exercises	.92
Development vs. Discussion : IM	.93
Development vs. Discussion : TM	.67

SUMMARY

Three creative tests were specially constructed for the present study (CG1, CMG and CG2), and new scoring procedures were developed to assess students' creative performance on these tests in terms of <u>fluency</u>, <u>diversity</u> and <u>rarity</u>. In order to achieve

objectivity and accuracy in our assessment, a method of establishing a set of significant Diversity Categories for each creative problem was postulated and applied to all the 6 creative items. The relevant content validity and construct validity of these three tests were found to be quite tenable, and their reliabilities high enough for the purposes of our study.

A traditional transfer test (TAR) was also developed to assess students' ability to apply learned concepts and skills in geometric transformation to conventional area-finding problem-situations. Two other traditional tests, SEC and TMG were selected. The validity and reliability of TAR, SEC and TMG were well-established. With regard to the other three traditional measures (GR7, A5T and MAR) obtained from students' records, there was satisfactory indication that these are valid scores reflecting students' mathematical achievement prior to the treatments.

Finally, the validity and reliability of the Observer Rating Scale of Teacher Behavior appear to be satisfactory for the objectives of the present investigation.

CHAPTER VII

ANALYSIS OF RESULTS

INTRODUCTION

The present study employed multiple measures to assess the effectiveness of the Inventive Method of instruction in junior high motion geometry. The data obtained provide the scientific basis for answering our three fundamental questions: (1) Did the teacher implement the two treatments (IM and TM) according to the experimental design? (2) Did students of the two groups differ initially in terms of factors relevant to the study? (3) Did these two groups perform differently as a result of the different treatment effects?

These questions were analysed in terms of derived hypotheses through appropriate statistical techniques. In our study, an intact class was randomly assigned to the treatment. Strictly speaking then it was the group (class) and not the student which was the "experimental unit". Hence, the group mean rather than an individual student score constituted the basic datum of our experiment (Airasian, 1974:181-182). On the other hand, statistical analyses in this study were performed using the number of students to determine the number of degrees of freedom available in the data. This resulted in less conservative critical values for hypotheses testing. Since ours is the first attempt to operationalize Boychuk's theoretical model, a more liberal criterion is preferable so as to encourage further exploration along the same line of research.

TEACHER'S CLASSROOM BEHAVIOR

In order to monitor the implementation of the two instructional methods, the investigator undertook 24 regular classroom observations to gather both qualitative and quantitative information about the teaching and learning activities in both groups. Qualitative information enabled the investigator to rectify any deviation from the designed procedures if necessary. Quantitative data collected through the _Observer Rating Scale of Teacher Behavior_ (Naciuk, 1968) provided an answer to the research questions pertinent to the actual implementation of the various phases of the instructional methods.

Qualitative Analysis

It was observed that students of both groups seemed to ignore the presence of the investigator at the back corner of the room after the first two visits. The teacher's observations confirm that students' behavior was not affected by the investigator's presence. Both treatment groups were informed of the main purposes of this experimental study, and questions invited from the students. The only query raised was the relevance of the learning materials to normal schoolwork. In this regard, the students were assured emphatically by the participating teacher that everything learned and all exercises assigned in the classroom were taken from the _Program_ designed by the school board, and thus their final evaluations would be based on these same learning materials.

This remark appeared to satisfy all the students of the two groups.

They were also told at the beginning of the treatments that the two different sets of exercises, IE and TE contained essentially the same mathematical content in somewhat different format. When they were asked at the end of the treatment period whether any of them had seen materials used in the other class, only one student of the TM group admitted that she had glanced out of curiosity through a few IE's of her friend. Students also did not seem to discuss mathematical exercises after class with their friends in the other class. These facts appear then to rule out the possibility of contamination effects in our study.

The participating teacher made full use of the two booklets provided for the two different instructional units and methods. No serious deviations from the planned materials and procedures were observed. However, more time was spent on the Discussion phase of instruction than expected, taking up at least 20 minutes and sometimes even the whole 36 to 37 minute period. On balance though, roughly the same amount of time was spent on classroom Discussion (ID and TD). The Development (DC) phase of both the IM and the TM groups was about 10 to 15 minutes for each lesson. About 95% of the Exercises (IE and TE) were assigned to students as homework. All exercises assigned were discussed in the respective classes according to the prescribed solutions or suggested solutions. All students did most of the exercises assigned.

No special discussion with the teacher was therefore needed to facilitate satisfactory implementation of the two instructional

methods. Three 20-minute short discussions, however were held to make some minor modifications to accommodate for the loss of two full instructional periods due to unexpected holidays. To ensure the completion of all 19 lessons, the last three lessons were thus taught in quicker and more directed instruction for both groups. Overall observation indicated that the two methods of instruction were implemented according to plan.

Quantitative Analysis

During the 24 classroom observations, the investigator also rated the teacher's classroom behavior on the Observer Rating Scale of Teacher Behavior. Such rating was performed for all 24 observations, 12 for each group. Within each IM or TM group, 6 ratings were made during the Development of Concept phase of instruction, and 6 during the respective Discussion phase (ID or TD). The purpose of these ratings was to collect objective and reliable data for analyses. Figure 7.1 shows profiles of teacher's classroom behavior for the four phases of instruction of the two methods, based on data from the Observer Rating Scale. The profiles indicate that the teacher exhibited a higher level of "inventive teaching" on the first 5 dimensions of the Rating Scale during the Inventive Discussion phase of the IM. Similar "expository teaching" seemed to prevail in the Development of Concept phase of both IM and TM, as well as the Traditional Discussion phase of the TM. These conclusions were verified

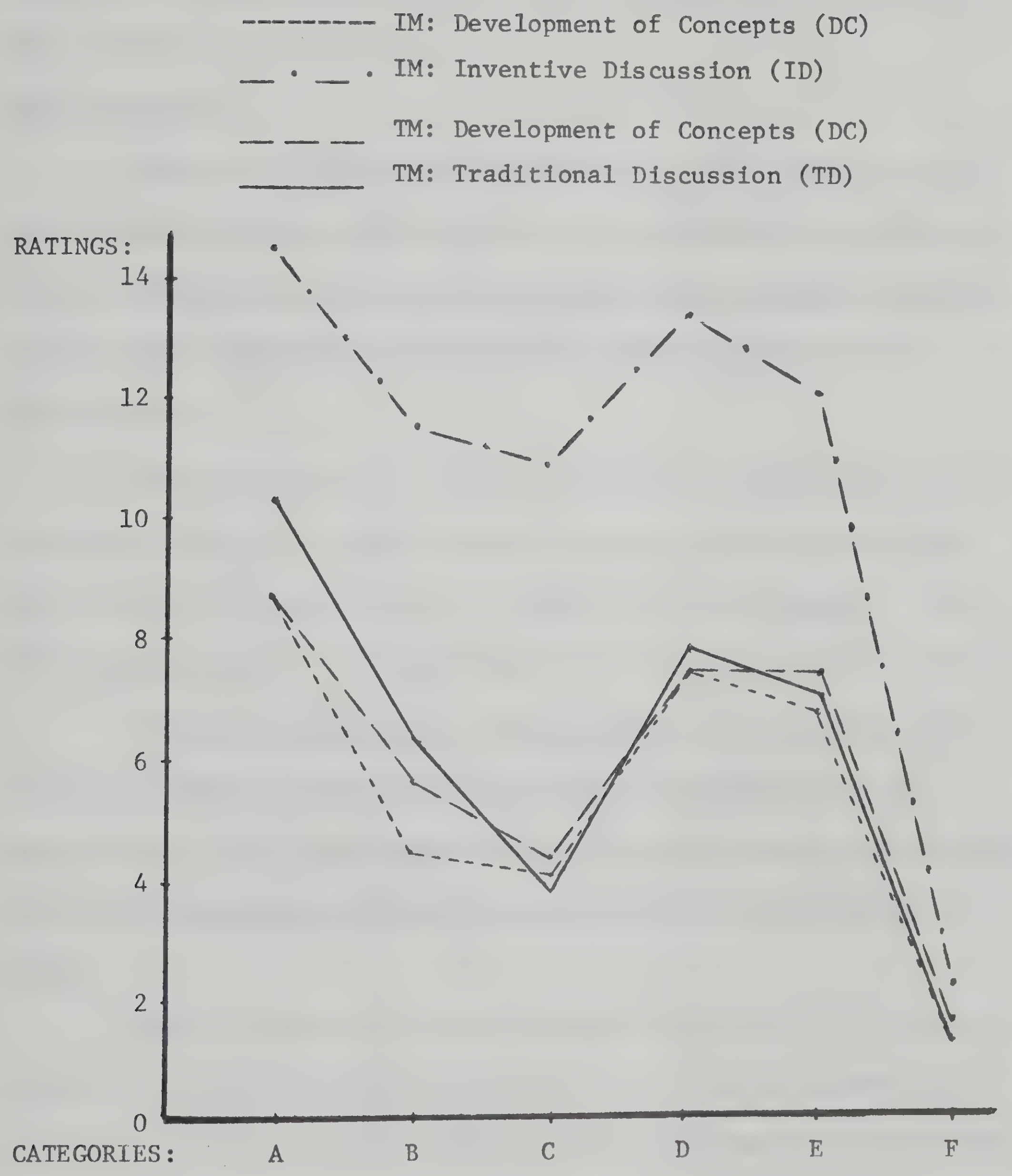


FIGURE 7.1 PROFILES OF TEACHER'S CLASSROOM BEHAVIOR BASED ON DATA FROM OBSERVER RATING SCALE ON 12 OBSERVATIONS ON EACH GROUP.

rigorously through performing appropriate Hotelling's T^2 Tests on the following null hypothesis.

Null Hypothesis 1:

There is no significant difference : (a) between the two instructional methods with respect to the Development of Concepts and the Discussion phases of instruction, and (b) between these two phases within each method, measured in terms of scores on the Rating Scale.

This hypothesis was further analysed in terms of four sub-hypotheses, which test the equality of population centroids which are vectors of ratings on the six dimensions of the Scale. These sub-hypotheses and the related significance tests are given below.

(a) Null Hypothesis 1a: With respect to the ratings of teacher's behavior during the Development of Concept phase of instruction, there is no significant difference between the centroids of the two populations from which the two groups (IM and TM) were drawn.

Table 7.1 shows that the computed Hotelling's T^2 - statistic is 9.05. The corresponding F-statistic of .75 is not significant at the .05 level. The Null Hypothesis 1a hence cannot be rejected. The centroids of the two populations from which our two groups were drawn are therefore equal, and we can conclude that both IM and TM groups were in fact exposed to quite similar content instruction in motion geometry.

TABLE 7.1

HOTELLING'S T^2-TEST ON SCORES OBTAINED FROM OBSERVER RATING SCALE FOR RESPECTIVE DEVELOPMENT OF CONCEPTS (DC) PHASE OF (IM) AND (TM) GROUPS.

	NO. OF OBSERVATIONS	CATEGORY						T^2	F(6,5)	P
		A	B	C	D	E	F			
IM: DEVELOPMENT OF CONCEPTS	6	8.83	4.67	4.17	7.33	7.33	1.67	9.05	.75	.63
TM: DEVELOPMENT OF CONCEPTS	6	8.83	5.67	4.50	7.33	6.50	1.67			

(b) Null Hypothesis 1b: With respect to the ratings of teacher's behavior during the Discussion phase (ID or TD) of instruction, there is no significant difference between the centroids of the two populations from which the two groups were drawn.

Table 7.2 shows that the corresponding F-statistic of 15.22 for the computed T^2 of 182.65 far exceeds the 95th centile of the F-distribution with 6 and 5 d.f.'s. The Null Hypothesis 1b is hence rejected at the .05 level. The two group centroids are therefore significantly different -- a conclusion that indicates that different classroom discussions were conducted in the two groups. The higher ratings for the ID phase of instruction on the six dimensions further indicates that a more inventive approach was in fact adopted during the ID in comparison with that implemented in the TD.

(c) Null Hypothesis 1c: With respect to the ratings of teacher's behavior within the two phases (DC and ID) of the IM, there is no significant difference between the centroids of the two populations from which these two samples were drawn.

The computed T^2 of 184.27 with the corresponding F-statistic of 15.36 presented in Table 7.3 is highly significant at the .05 level. The Null Hypothesis 1c is therefore rejected, i.e. the two population centroids are significantly different at the 5% level. Coupled with the higher ratings for the ID phase, we thus conclude that an inventive approach was implemented in the ID phase of the IM, when compared with that of the DC phase within the IM group.

TABLE 7.2

HOTELLING'S T^2-TEST ON SCORES OBTAINED FROM OBSERVER RATING SCALE FOR INVENTIVE DISCUSSION AND TRADITIONAL DISCUSSION.

	NO. OF OBSERVATIONS	CATEGORY						T^2	F(6,5)	P
		A	B	C	D	E	F			
INVENTIVE DISCUSSION	6	14.33	11.33	10.83	13.17	12.00	2.5	182.65	15.22	.0045
TRADITIONAL DISCUSSION	6	10.33	6.17	4.00	7.67	6.83	1.5			

TABLE 7.3

HOTELLING'S T^2-TEST ON SCORES OBTAINED FROM THE OBSERVER RATING SCALE FOR DEVELOPMENT AND DISCUSSION PHASES OF (IM) GROUP.

	NO. OF OBSERVATIONS	CATEGORY						T^2	F(6,5)	P
		A	B	C	D	E	F			
DEVELOPMENT OF CONCEPTS (IM)	6	8.83	4.67	4.17	7.33	7.33	1.67	184.27	15.36	.0044
INVENTIVE DISCUSSION	6	14.33	11.33	10.83	13.17	12.00	2.50			

(d) <u>Null Hypothesis 1d</u>: <u>With respect to the ratings of teacher's behavior within the two phases (DC and TD) of the TM, there is no significant difference between the centroids of the two populations from which these two samples were drawn</u>.

The results presented in Table 7.4 show that the T^2 of 3.61 with the corresponding F-statistic of .30 falls far short of the 95th centile of the F-distribution with 6 and 5 d.f.'s. Thus the Null Hypothesis 1d cannot be rejected at the .05 level. The teacher's behavior therefore did not differ significantly during the two phases, DC and TD of the TM instruction. The low ratings for both DC and TD of the TM indicate that a traditional expository approach of teaching was adopted throughout the TM instruction.

In short, Hotelling's T^2 tests clearly demonstrate the accurate implementation of the two instructional methods as prescribed. Similar Development of Concepts (DC) phases were employed in both IM and TM groups. An inventive approach was exhibited in the Inventive Discussion (ID) phase of the IM group, either in comparison with the DC phase of the IM, or the Traditional Discussion (TD) of the TM group. However, the DC and TD phases of the TM group exhibited similar traditional types of expository instruction. Therefore, the two groups of students were actually exposed to the kind of learning experience designed for the respective groups -- a conclusion which justifies the following analyses of the entrance and terminal behavior of these students to assess the effectiveness of the constructed inventive method of teaching grade eight motion geometry.

TABLE 7.4

HOTELLING'S T^2-TEST ON SCORES OBTAINED FROM OBSERVER RATING SCALE FOR DEVELOPMENT AND DISCUSSION PHASES OF (TM) GROUP.

	NO. OF OBSERVATIONS	CATEGORY						T^2	F(6,5)	P
		A	B	C	D	E	F			
DEVELOPMENT OF CONCEPTS (TM)	6	8.33	5.67	4.50	7.33	6.50	1.67	3.61	.30	.91
TRADITIONAL DISCUSSION	6	10.33	6.17	4.00	7.67	6.83	1.50			

STUDENTS' ENTRANCE BEHAVIOR

Due to the inflexibility of the school schedule, random assignment of individual students to treatments was not feasible. Intact classes, however, were assigned to the two instructional methods randomly. There were 30 students in the experimental group and 28 students in the control group. After eliminating students who were absent from one or more of the 9 tests (5 pretests and 4 posttests), complete data were found to be obtained for 41 students, 20 for the experimental group (IM group), and 21 for the control group (TM group)[1]. Table 7.5 summarizes the number of girls and boys involved in the final sample. The ages of the IM group ranged from 13.1 years to 13.8 years with a mean of 13.4 years; while that of the TM group ranged from 12.2 years to 14 years, with a mean of 13.5 years.

TABLE 7.5

NUMBER OF STUDENTS IN THE INVENTIVE AND TRADITIONAL GROUPS

	EXPERIMENTAL (Inventive Method)	CONTROL (Traditional Method)
Girls	12	10
Boys	8	11
TOTAL	20	21

[1]Statistical analyses testing the differences between the two groups including students without complete data are given in Appendix I.

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
	[illegible]
	[illegible]

[illegible]

According to the school principal, students of the school are distributed across classes at grade seven in a more or less random manner, without special groupings procedure. Information on parental occupations furthermore, indicate a fairly homogeneous working-class background. It therefore seems reasonable to assume that the two groups are more or less equivalent with respect to socio-economic background, as well as intellectual ability and academic achievement. Nevertheless, we took the additional precaution of statistically assessing the differences between the two groups on three measures of mathematical background (GR7, A5T and MAR), and two measures of mental capacity (SEC and CG1). Table 7.6 shows the means and standard deviations of the two groups on these five measures (pretests). Figure 7.2 shows the corresponding profiles of the two groups based on these pretest means. The Inventive group (IM) appeared to score slightly better on GR7, A5T, MAR and SEC, while the Traditional group (TM) seemed to perform better on the CG1. The significance of the differences on all the pretest means considered simultaneously was tested by computing the Hotelling's T^2-statistic. The corresponding null hypothesis tested was:

Null Hypothesis 2:

With respect to the students' scores on the 5 pretests (GR7, A5T, MAR, SEC and CG1), there is no significant difference between the centroids of the two populations from which the two groups (IM and TM) were drawn.

TABLE 7.6

PRETEST MEANS AND STANDARD DEVIATIONS FOR INVENTIVE (IM) AND TRADITIONAL (TM) GROUPS:

		GR7		A5T		MAR		SEC		CG1					
										Fluency		Diversity		Rarity	
GROUP	N	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
INVENTIVE	20	68.7	14.5	64.1	21.8	69.9	20.1	21.6	4.6	28.6	6.8	18.8	3.5	10.7	8.8
TRADITIONAL	21	65.6	19.9	64.0	25.0	66.4	23.3	18.8	6.7	27.7	9.7	19.2	3.0	11.4	6.0

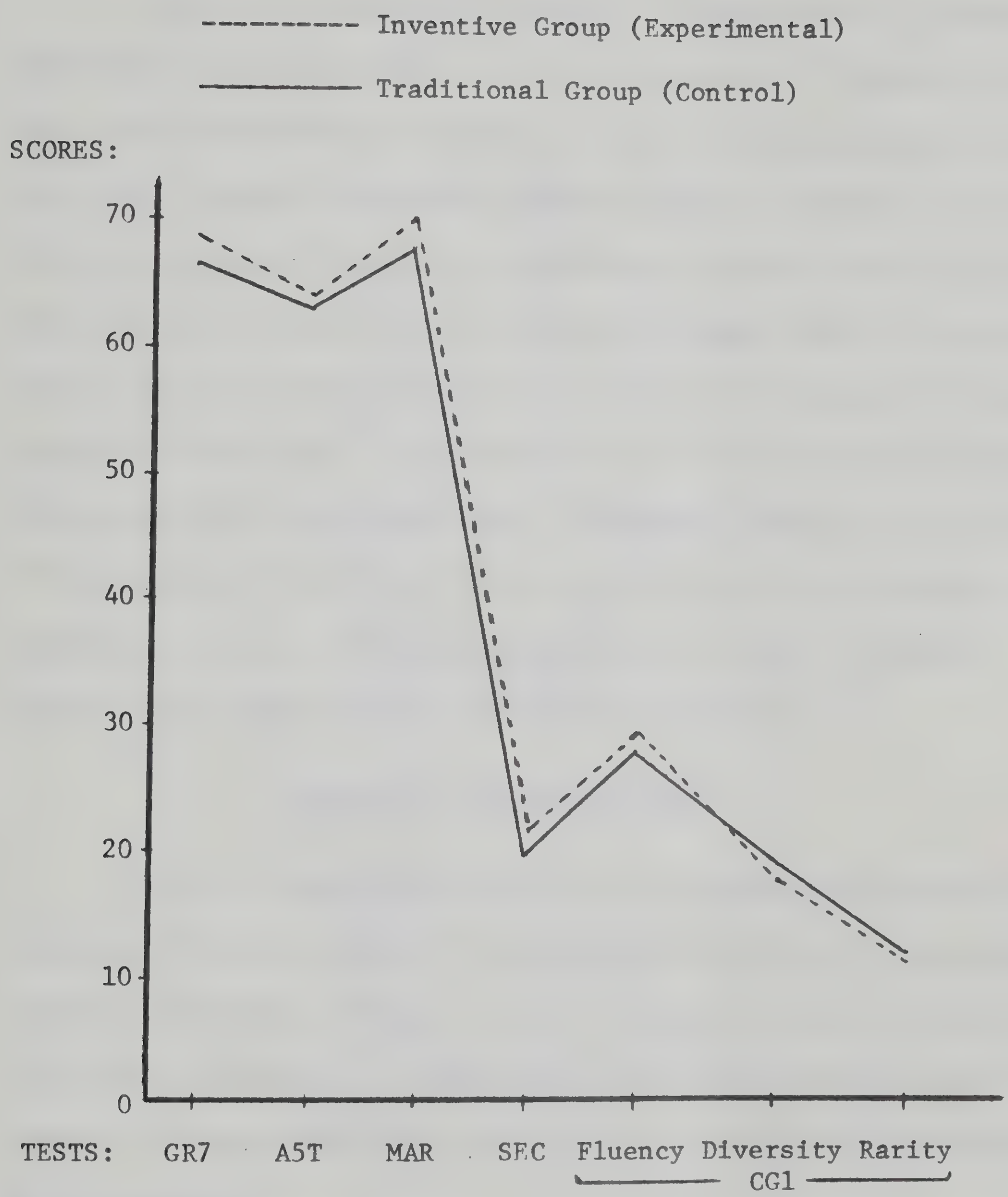


FIGURE 7.2 PROFILES OF PRETEST MEANS FOR THE INVENTIVE (IM) AND TRADITIONAL (TM) GROUPS

The results presented in Table 7.7 show that the assumption of homoscedasticity was tenable, while the computed T^2-statistic of 4.64 with the corresponding F-statistic of .56, is not significant at the .05 level. The Null Hypothesis 2 hence cannot be rejected. We conclude that the centroids of the two populations from which the IM and TM groups were drawn are equal; i.e. the two groups were equivalent with respect to the 5 pretest measures. Specifically, students of the two groups did not differ significantly in terms of their mathematical background, their Euclidean level of geometric maturity and their level of mathematical creativity. We can therefore proceed to analyse the outcomes of the two instructional methods in terms of students' terminal performance measured by the four posttests.

STUDENTS' TERMINAL BEHAVIOR

Table 7.8 summarizes the means and standard deviations of the two groups on the 4 posttests: TMG, TAR, CMG and CG2. The corresponding profiles indicated in Figure 7.3 show that the IM group scored higher on all the posttests. To compare the overall performance of the two groups, a one-way MANOVA was performed to test the Null Hypothesis 3:

Null Hypothesis 3:

With respect to students' scores on the 4 posttests (TMG, TAR, CMG and CG2), there is no significant difference between the centroids of the two populations from which the two groups (IM and TM) were drawn.

The results of the MANOVA given below show that the assumption of homoscedasticity was tenable, while the F-statistic of 8.06 is significant at the .05 level. Hence Null Hypothesis 3 is rejected, i.e.

TABLE 7.7

HOTELLING'S T^2-TEST ON MEANS OF ALL 5 PRETESTS: GRADE 7 MATH. (GR7), AVE. OF 5 TESTS (A5T), METRIC AREA TEST (MAR), PIAGETIAN SECTIONING TEST (SEC) AND CREATIVE GEOMETRY TEST I (CG1).
(N=41)

GROUP	N	GR7	A5T	MAR	SEC	CG1			T^2	F(7,33)	P
						Fluency	Diversity	Rarity			
INVENTIVE	20	68.7	64.1	69.9	21.6	28.6	18.8	10.7	4.64	.56	.78
TRADITIONAL	21	65.6	64.0	66.4	18.8	27.7	19.2	11.4			

Bartlett-Box Homogeneity of Variance-Covariance Tests:

DF = 28, χ^2 = 35.12, P = .166

F(28, 5271.5) = 1.25, P = .174

TABLE 7.8

POSTTEST MEANS AND STANDARD DEVIATIONS FOR INVENTIVE (IM) AND TRADITIONAL (TM) GROUPS:

		TMG		TAR		CMG						CG2					
						Fluency		Diversity		Rarity		Fluency		Diversity		Rarity	
GROUP	N	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
INVENTIVE	20	26.6	7.2	33.1	7.9	19.5	4.5	28.4	4.5	29.2	15.5	16.1	7.0	22.9	4.5	30.1	14.1
TRADITIONAL	21	25.5	8.0	21.5	8.0	12.5	4.7	22.3	5.6	17.4	12.9	11.9	4.1	17.7	2.5	14.4	7.2

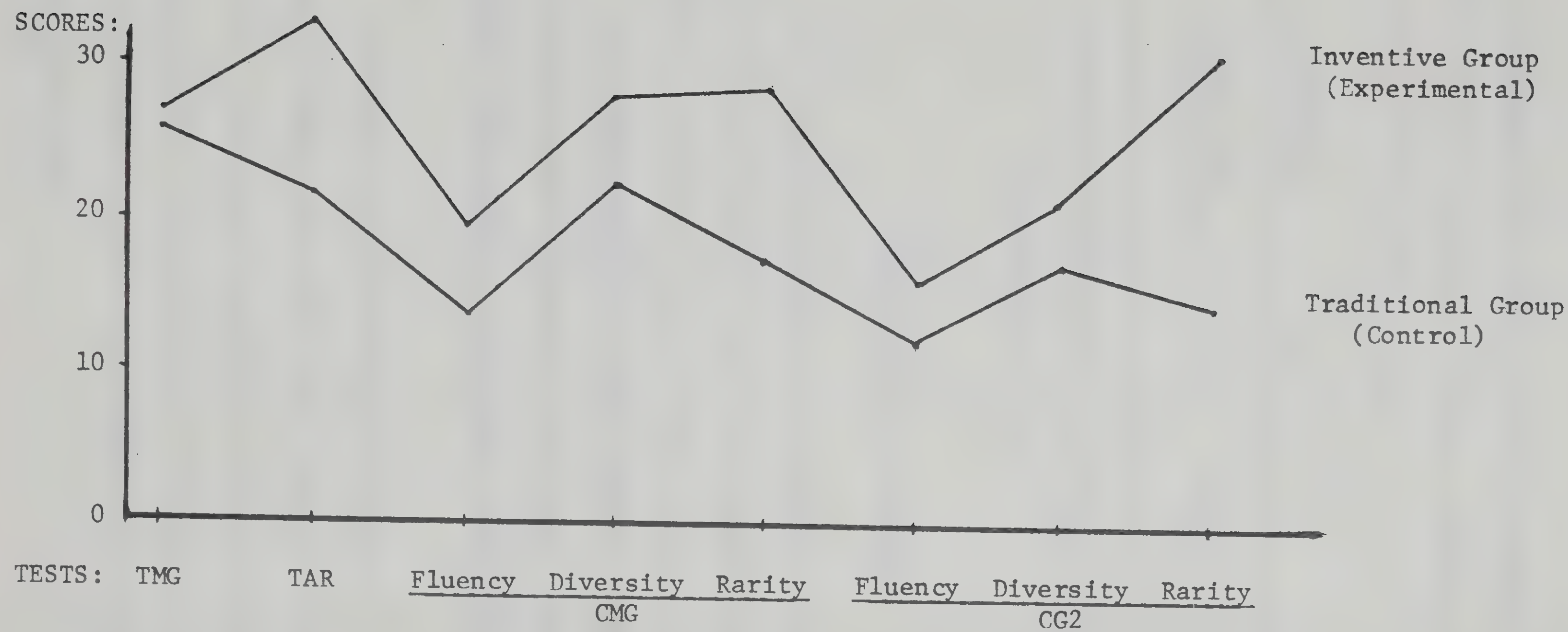
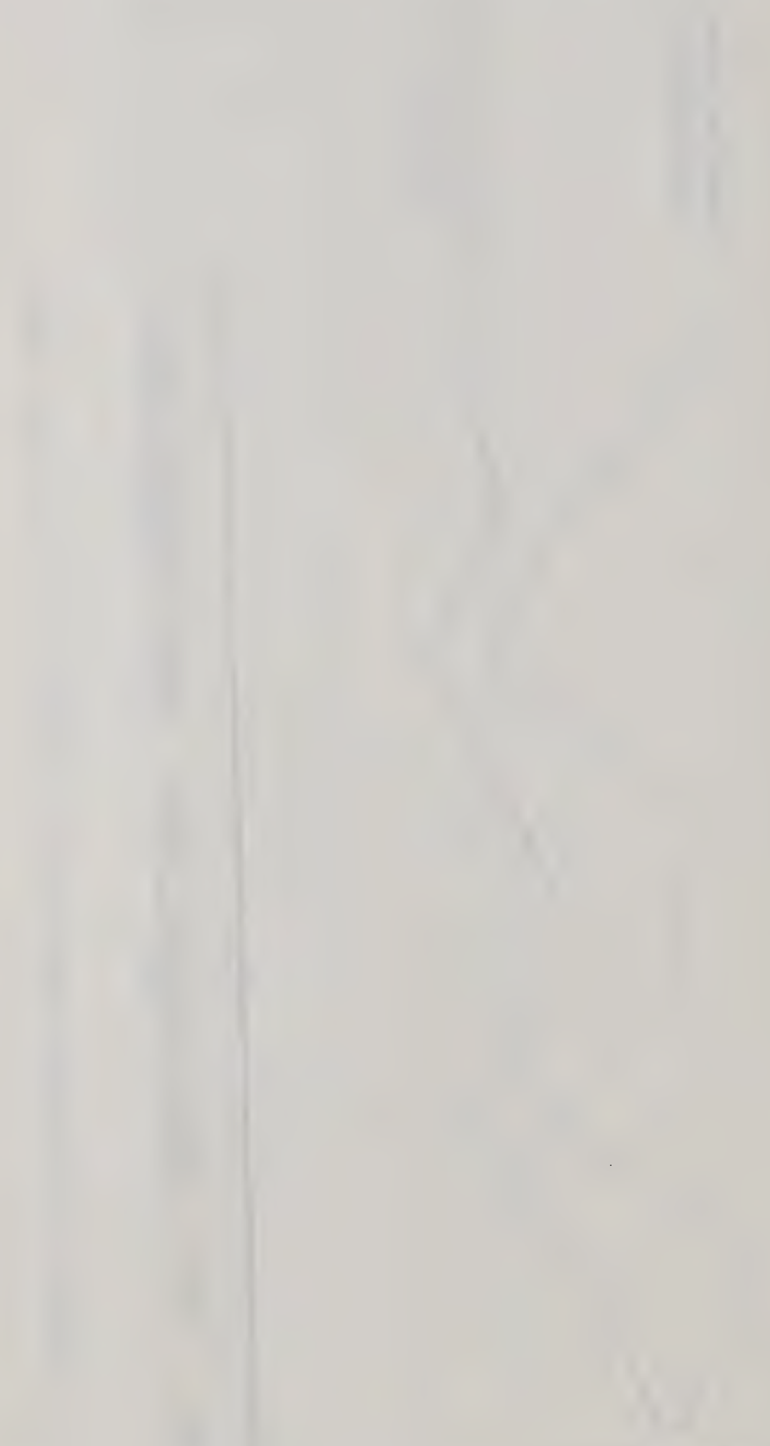


FIGURE 7.3 PROFILES OF THE POSTTEST MEANS FOR THE INVENTIVE AND TRADITIONAL GROUPS.



the IM treatment resulted in better performance. These results justified further analyses to answer the specific research questions postulated for the present study.

VARIABLES	DF	F	P	WILK'S λ
TMG, TAR CMG and CG2	8, 32	8.06	.00001	.332

Bartlett-Box Homogeneity of Variance-Covariance Test:

$F(36, 5090.6) = 1.42, \qquad P > .05$

Traditional Achievement

Null Hypothesis 3a was postulated to assess the difference between the performance of the two groups of students in solving traditional convergent type of motion geometry problems.

Null Hypothesis 3a: With respect to students' scores on the Traditional Motion Geometry Test (TMG), the means of the two populations from which the two groups (IM and TM) were drawn, are equal.

A simple one-way ANOVA was performed. The results presented in Table 7.9 show that the F ratio of .22 is not significant at the .05 level. Hence, Null Hypothesis 3a cannot be rejected. We therefore conclude that the Inventive group (IM) and the Traditional group (TM) attained the same level of achievement on the Traditional Motion Geometry Test (TMG).

TABLE 7.9

ANALYSIS OF VARIANCE (ANOVA) FOR SCORES ON TRADITIONAL MOTION GEOMETRY TEST (TMG).
(N=41)

SOURCE	DF	SS	MS	F	P
TREATMENT	1	12.94	12.94	.22	.639
ERROR	39	2256.04	57.85		

Bartlett Homogeneity of Variance Test:

$\chi^2 = .190$, P = .663

TABLE 7.10

ANALYSIS OF COVARIANCE (ANCOVA) FOR SCORES ON TRADITIONAL MOTION GEOMETRY TEST (TMG), USING AS COVARIANCES PRETEST SCORES ON GRADE 7 MATH. (GR7), AVE. OF 5 TESTS (A5T), METRIC AREA TEST (MAR), PIAGETIAN SECTIONING TEST (SEC) AND CREATIVE GEOMETRY TEST I (CG1). (N=41)

SOURCE	DF	SS	MS	F	P
TREATMENT	1	1.00	1.00	.05	.832
ERROR	32	699.11	21.85		

Bartlett-Box Tests of Homogeneity of Variance:

DF = 1, χ^2 = .012, P = .913
F(1, 1867.01) = .012, P = .913

Multivariate Tests of Parallelism:

F(7, 25) = 1.034, P = .433, Wilk's λ = .775

Though our analysis of the students' entrance behavior of the two groups did not detect any significant differences between the two groups with respect to students' performance on the five pretests, we argued for the need to partial out that portion of the treatment effects attributable to students' initial level of mathematical knowledge and mental capacity (supra: 131). Students' scores on the criterion variable (TMG) were therefore further analysed using one-way ANCOVA with the 5 pretests as covariates. Table 7.10 summarizes the results of the analysis on the "adjusted" mean scores on TMG. While the assumptions of homoscedasticity and parallelism were tenable, it was found that the two groups did not differ significantly at the .05 level.

Creative Achievement

Null Hypothesis 3b was postulated to assess the difference between the performance of the two groups of students in solving inventive divergent types of motion geometry problems.

Null Hypothesis 3b: With respect to students' scores on the Creative Motion Geometry Test (CMG), the centroids of the two populations from which the two groups (IM and TM) were drawn, are equal.

Since the centroids of the groups are defined in terms of three variables (fluency, diversity and rarity scores), one-way MANOVA was performed to detect the significance of treatment effects. Inspection of Table 7.11 shows that the assumption of homoscedasticity was not tenable at the 5% level. Thus, the

TABLE 7.11

MULTIVARIATE ANALYSIS OF VARIANCE (MANOVA) OF SCORES ON CREATIVE MOTION GEOMETRY TEST (CMG).
(N=41)

SOURCE	SSP			F(3, 37)	P	WILK'S λ
TREATMENT	498.20					
	429.83	370.83		9.08	.00012	.576
	874.79	754.73	1536.03			
ERROR	832.19					
	770.52	1043.22				
	2046.61	2642.62	7920.36			

Bartlett-Box Homogeneity of Variance-Covariance Tests:

DF = 6, χ^2 = 22.20, P = .001

F(6, 10942.8) = 3.70, P = .001

TABLE 7.12

MULTIVARIATE ANALYSIS OF COVARIANCE (MANCOVA) FOR SCORES ON CREATIVE MOTION GEOMETRY TEST (CMG), USING AS COVARIATES PRETEST SCORES ON GRADE 7 MATH. (GR7), AVE. OF 5 TESTS (A5T), METRIC AREA TEST (MAR), PIAGETIAN SECTIONING TEST (SEC), AND CREATIVE GEOMETRY TEST I (CG1).
(N=41)

SOURCE	SSP			F(3, 30)	P	WILK'S λ
TREATMENT	383.44			11.62	.00003	.462
	313.71	256.66				
	636.49	520.74	1056.52			
ERROR	505.77					
	454.84	696.69				
	1414.39	1910.83	6260.73			

Bartlett-Box Homogeneity of Variance-Covariance Tests:

DF = 6, χ^2 = 9.73, P = .14

F(6, 4451.7) = 1.62 P = .14

Multivariate Tests of Parallelism:

F(21, 66.6) = .69, P = .83, Wilk's λ = .568

multivariate test, though it appears highly significant, may not be valid. However, when a MANCOVA was carried out, to partial out the possible effects of the factors measured by the 5 pretests, the significant treatment effects presented in Table 7.12 are supported by the validity of the two basic assumptions of homoscedasticity ($P > .05$) and parallelism ($p > .05$). We therefore conclude that the Inventive group performed significantly better than the Traditional group on the <u>Creative Motion Geometry Test (CMG)</u>.

Transfer of Learning

The question of <u>transferability</u> was examined from two aspects: (1) transfer to inventive-divergent geometrical problem-situations, and (2) transfer to traditional-convergent geometrical problem-situations. For these purposes, two tests were constructed: (1) <u>Creative Geometry Test II (CG2)</u>, and (2) <u>Transfer Area Test (TAR)</u>. Two corresponding null hypotheses were therefore formulated to test the treatment effects on students' performance on these two tests.

(1) <u>Creative Geometry Test II (CG2)</u>

<u>Null Hypothesis 3c</u>: <u>With respect to students' scores on CG2, the centroids of the two populations from which the two groups (IM and TM) were drawn, are equal</u>.

The MANOVA results presented in Table 7.13 show that the F-statistic of 7.05 is significant at the .05 level. Since the basic assumption of homosedasticity was tenable, the null hypothesis is rejected. The conclusion that the Inventive group performed

TABLE 7.13

MULTIVARIATE ANALYSIS OF VARIANCE (MANOVA) OF SCORES ON CREATIVE GEOMETRY TEST II (CG2).
(N=41)

SOURCE	SSP	F(3, 37)	P	WILK'S λ
TREATMENT	$\begin{bmatrix} 176.02 & & \\ 220.20 & 275.48 & \\ 667.48 & 835.03 & 2531.15 \end{bmatrix}$	7.05	.00073	.636
ERROR	$\begin{bmatrix} 1262.76 & & \\ 552.53 & 514.09 & \\ 1658.66 & 1525.49 & 4826.75 \end{bmatrix}$			

Bartlett-Box Homogeneity of Variance-Covariance Tests:

DF = 6, $\chi^2 = 11.28$, P = .080

F(6, 10942.8) = 1.88, P = .080

TABLE 7.14

MULTIVARIATE ANALYSIS OF COVARIANCE (MANCOVA) FOR SCORES ON CREATIVE GEOMETRY TEST II (CG2), USING AS COVARIATES PRETEST SCORES ON GRADE 7 MATH. (GR7), AVE. OF 5 TESTS (A5T), METRIC AREA TEST (MAR), PIAGETIAN SECTIONING TEST (SEC), AND CREATIVE GEOMETRY TEST I (CG1).
(N=41)

SOURCE	SSP	F(3, 30)	P	WILK'S λ
TREATMENT	$\begin{bmatrix} 175.36 & & \\ 215.42 & 264.62 & \\ 646.16 & 794.98 & 2388.28 \end{bmatrix}$	14.06	.00001	.416
ERROR	$\begin{bmatrix} 612.18 & & \\ 158.57 & 193.61 & \\ 471.43 & 545.43 & 1777.43 \end{bmatrix}$			

Bartlett-Box Homogeneity of Variance-Covariance Tests:

DF = 6, χ^2 = 7.85, P = .25

F(6, 4451.7) = 1.31 P = .25

Multivariate Tests of Parallelism:

F(21, 66.6) = 1.10, P = .37, Wilk's λ = .425

better on CG2 than the Traditional group is thus warranted. Similarly, the results of Table 7.14 indicate the significance of the MANCOVA test at the .05 level, and that the two basic assumptions were tenable. The superior performance of the Inventive group on the Creative Geometry Test (CG2) over that of the Traditional group can thus be confirmed with confidence.

(2) Transfer Area Test (TAR)

Null Hypothesis 3d: With respect to students' scores on TAR, the means of the two populations from which the two groups (IM and TM) were drawn, are equal.

Table 7.15 presents the results of the one-way ANOVA test. The difference between the two means was highly significant at the .05 level. A similar result was obtained through the one-way ANCOVA test (Table 7.16) to account for the possible effects of the 5 pretest measures. The two basic statistical assumptions of homoscedasticity and parallelism were again tenable. We can therefore conclude confidently that the Inventive group scored better than the Traditional group on the Transfer Area Test (TAR).

TABLE 7.15

ANALYSIS OF VARIANCE (ANOVA) FOR SCORES ON TRANSFER AREA TEST (TAR). (N=41)

SOURCE	DF	SS	MS	F	P
TREATMENT	1	1384.09	1384.09	21.81	.000036
ERROR	39	2475.04	63.46		

Bartlett Homogeneity of Variance Test:

$\chi^2 = .004$, P = .95

TABLE 7.16

ANALYSIS OF COVARIANCE (ANCOVA) FOR SCORES ON TRANSFER AREA TEST (TAR), USING AS COVARIATES PRETEST SCORES ON GRADE 7 MATH. (GR7), AVE. OF 5 TESTS (A5T), METRIC AREA TEST (MAR), PIAGETIAN SECTIONING TEST (SEC) AND CREATIVE GEOMETRY TEST I (CG1).
(N=41)

SOURCE	DF	SS	MS	F	P
TREATMENT	1	926.16	926.16	25.41	.00002
ERROR	32	1166.60	36.46		

Bartlett-Box Tests of Homogeneity of Variances:

DF = 1 χ^2 = 1.835, P = .176

F(1, 1867.01) = 1.836, P = .176

Multivariate Tests of Parallelism:

F(7, 25) = 1.940, P = .105, Wilk's λ = .648

SUMMARY

The evaluation phase of the present study proceeded along three dimensions.

First, the investigator undertook 24 classroom observations to monitor the implementation of the two instructional methods and obtain information on the participating teacher's classroom behavior. The analysis of the qualitative information indicated quite accurate implementation of the prescribed instructional procedures and materials. This conclusion was further confirmed by the statistical analyses of the quantitative data collected through the Observer Rating Scale.

Secondly, to confirm that the two groups of students (experimental n=20, control n=21) actually come from equivalent populations, empirical data were obtained on 5 measures (GR7, A5T, MAR, SEC and CG1), which were considered to be adequate indicators of the relevant background of these 41 students. The multivariate tests on these 5 pretest scores detected no significant difference between the two groups with respect to the students' mathematical background, their level of geometric maturity, and level of mathematical creativity.

Finally, the effects of the two instructional methods were evaluated on the basis of students' performance on the 4 posttests: TMG, CMG, CG2 and TAR. The results of significance tests are summarized as follows:

(1) The Inventive (IM) group and the Traditional group

(TM) performed at the same level of achievement on the Traditional Motion Geometry Test (TMG).

(2) The Inventive group scored significantly higher than the Traditional group on the Creative Motion Geometry Test (CMG), Creative Geometry Test II (CG2), and Transfer Area Test (TAR).

Students' initial mathematical competency and mental ability did not seem to affect the direction of the above results. These results hold true even when the group means (centroids) were adjusted to account for the possible effects of students' initial level of achievement and ability. In effect, the results show that the constructed Inventive Method (IM) of instruction in grade eight motion geometry can maintain the students' competency in the content matter, and at the same time enhance their mathematical creativity. Moreover, IM enables them to transfer creative learning experience and learned knowledge to related geometrical problem-situations better than their counterparts taught under the TM of instruction.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

INTRODUCTION

Among the voluminous literature on creativity (Treffinger and Poggio, 1972: 253), there appears to be relatively few studies dealing directly with creative behavior in mathematical problem-situations (Kilpatrick, 1969: 166). The present study was thus prompted by the observation that the "creative spirit is almost completely ignored" in school mathematics (Ailles, Norton and Steel, 1973: 7), although "fostering independent and creative thinking" should be of prime concern in mathematics education (Cambridge Conference on School Mathematics, 1963: 17).

Most studies of mathematical creativity have been concerned with the construction of measuring instruments to identify postulated creative processes or products in mathematical situations (Kilpatrick, 1969: 166). These studies appear to substantiate the belief that mathematical creativity does exist and is measurable in some meaningful way. Clearly, a next important question is how we can encourage creativity at the classroom level. The answer to this question involves such factors as "the conditions or situations, the practices or experiences, the approaches and attitudes that are conducive to

the production of novel, appropriate, quality ideas" (Crockenberg, 1972: 43). Our study is an exploratory attempt to furnish such an answer in junior high school mathematics.

SUMMARY OF RESULTS

The related literature on creativity reveals the pervasiveness of "many difficult problems which have not been solved" (Treffinger and Poggio, 1972: 253). Though most of these are theoretical issues dealing with the definition and measurement of creativity, "the need of a firmer theoretical foundation" for curriculum study such as ours (Dessart and Frandsen, 1973: 1191) necessitated the consideration of these issues along four major dimensions: (1) the theoretical or conceptual description of creativity (Treffinger, Renzulli and Feldhusen, 1971: 105), (2) the operational definition and criteria of assessment (Treffinger, Renzulli and Feldhusen, 1971: 107; Crockenberg, 1972: 40-43), (3) the content matter for creative thinking (Johnson and Kidder, 1972), and (4) the creative instruction or training (Torrance, 1972). These four dimensions thus constituted the conceptual framework of the present study.

Four major objectives were set for the investigation: (1) to identify a valid psychological model of creative problem solving in school mathematics with appropriate adaptation to junior high mathematics curriculum; (2) to operationalize the adapted conceptual model in terms of concepts derived from related

instructional theory, so as to identify significant common parameters underlying problem-situations reflecting the creative processes defined by the model, and thus provide practical guidelines for developing instructional method and materials that would promote students' mathematical creativity; (3) to select, from the junior high mathematics curriculum, an appropriate content domain which is conducive to creative thinking, and to design a creative approach to instruction and instructional materials; and (4) to evaluate the effectiveness of the instructional method and materials through an evaluation design with proper experimental procedures, instrumentation and statistical analyses.

The following sections give a brief overview of the results obtained with respect to these four purposes of the study.

The Three-Process Model

Our analysis of relevant works on creativity in general and mathematical creativity in particular, showed that Boychuk's (1974) psychological model provides a valid and meaningful description of students' creative behavior in mathematical problem-situations. The convergent process of "verification" was found to involve a "hypothesis-construction" aspect which could be adequately subsumed within the process of sensitivity, redefinition and conjecturing, and a "proof-construction" aspect which could be reasonably deemed beyond the resources of junior high school students. In the present study, mathematical

creativity was therefore defined conceptually in terms of the three creative problem solving processes of sensitivity, redefinition and conjecturing. It was argued that this Three-Process Model appears to be consistent with the needed "theoretical foundation" defined by Dessart and Frandsen (1973: 1191). Upon this foundation, meaningful instruction could be developed with appropriate instructional materials to facilitate the functioning of these three creative processes in mathematical problem-situations.

Operationalization of the Model

It has been shown that the discrepancy issue arises when a researcher imposes a conceptual definition of creativity on what was meant to be a narrow operational definition without clear indication of the logical connection between the two definitions. Obviously, this issue had to be resolved in order to translate our three-process model adequately into classroom instruction. We first established the fact that problem solving is a form of rule learning which can be properly described in terms of three parameters: domain (D), operation (O) and range (R). Creative mathematical problem-situations which optimally reflect the functioning of the three creative processes were then analysed in terms of these three parameters. Subsequently, three types of creative problem-situations were identified as prototypes of mathematical situations that optimally reflect the functioning of the creative processes of sensitivity, redefinition and

conjecturing. It was further found that the functioning of any one of these processes in any such situation cannot be identified exclusively. The common characteristics of fluency, diversity and rarity of ideas exhibited in the divergent productions (responses) to these situations, however, can be objectively assessed. For the purposes of the present study, mathematical creativity was therefore defined operationally in terms of fluency, diversity and rarity of ideas. In this way we established the necessary connection between our conceptual and operational definitions of mathematical creativity. These analyses further justified the use of our operational definition as criteria for measuring students' creative behavior in mathematical problem-situations.

Inventive Method of Instruction

Psychological theory of learning and development describes how students learn, under what conditions and how their mental abilities develop, while instructional theory prescribes how teaching can promote learning on the part of students and under what conditions (Bruner, 1971: 40). A psychological theory is meaningful in education only if it has practical implications for school learning. An instructional theory, however, can only be effectively implemented within the framework of a certain psychological theory about learning. In the present study, the psychological theory adopted was the creative problem solving model postulated by Boychuk (1974). The three creative processes

defined in this theoretical model were shown to describe adequately students' creative behavior in mathematical problem-situations, hence providing a valid psychological framework for implementing creative problem solving curriculum at the classroom level.

Secondly, we were faced with the task of developing instructional methods and materials which would effectively stimulate and promote creative thinking utilizing the three creative processes. A brief review of relevant literature was initially conducted to identify important dimensions and variables of instruction. Effective instructional strategies and approaches were formulated by focusing on the two basic functions (display and control) of instruction, and manipulating variables along these two dimensions. In our study, the "display" dimension was kept constant for both the experimental and the control groups by adopting a direct expository approach of teaching. This formed the Development of Concept (DC) phase of instruction for both groups. The main objective of this arrangement was to ensure optimal mastery of identical subject content as prerequisite for solving both inventive divergent and traditional convergent types of mathematical problems. Experimental manipulation was therefore applied only to variables belonging to the "control" dimension.

We were aware, of course, that environmental, attitudinal and other psychological factors may make a difference in students' creative performance. For the sake of experimental control, however, as well as drawing justification from studies on creative

instruction and training (e.g. "discovery method" and "mathematizing mode"), "encouraging inventive divergent thinking" and "practice in inventive problem-situations" were selected as the two key components for creative teaching. These two "creative components" of instruction focused on both the conceptual and operational definitions of mathematical creativity, since (a) the constructed Inventive Exercises (IE) are problem-situations which call upon the functioning of the creative processes of sensitivity, redefinition and conjecturing, and (b) the students' responses to these situations would exhibit the characteristics of fluency, diversity and rarity of ideas. Consequently, two different instructional methods (IM and TM) were constructed.

Both the Inventive Method (IM) and the Traditional Method (TM) consisted of a sequence of instructions: Development of Concepts (DC), Exercises (IE or TE) and Discussion (ID or TD). Identical DC phases of instruction for the experimental (IM) and the control (TM) groups were implemented through an expository approach outlined in the Program (Edmonton Separate School Board, 1975). Special sets of Inventive Exercises (IE) were constructed to include both the three types of inventive divergent problem-situations, as well as traditional convergent situations. The Traditional Exercises (TE) consisted of only the latter type of problems provided in the Program. Both IE and TE were assigned to students as homework, and followed up with a class discussion (respectively ID and TD). Since junior high motion geometry with

its inventive, informal and dynamic features lends itself admirably to creative instruction, 19 topics on motion geometry were selected from the Program. Nineteen sets of IE were constructed for the IM treatment.

Measuring Instruments and Scoring Procedures

To evaluate adequately the effectiveness of the constructed Inventive Method (IM) of instruction, several measuring instruments were employed to collect relevant data for analyses. Measures were first obtained of students' entrance behavior on 5 pretests: (1) Grade 7 Mathematics Achievement (GR7), (2) Grade Point Average of 5 Tests (A5T), (3) Metric Area Test (MAR), (4) Piagetian Sectioning Test (SEC), and (5) Creative Geometry Test I (CG1). These measurements allowed us to determine whether the experimental and the control groups were equivalent with respect to mathematical background (GR7, A5T and MAR), Euclidean level of geometric maturity (SEC), and mathematical creativity (CG1). The Observer Rating Scale of Teacher Behavior was used to collect data on the teaching-learning activities of the various phases of the two instructional methods, so as to assess the adequacy of the implementation of these two methods (IM and TM). Finally, measures were obtained of students' terminal behavior on 4 posttests: (1) Traditional Motion Geometry Test (TMG), (2) Creative Motion Geometry Test (CMG), (3) Creative Geometry Test II (CG2), and (4) Transfer Area Test (TAR). Scores on GR7, A5T and MAR were obtained from

students' school records. SEC, TMG and the Observer Rating Scale are instruments which have been constructed and validated by other researchers. CG1, CMG, CG2 and TAR were specially designed by the investigator for the purposes of the present study.

Since the main objective of the IM was to stimulate and promote students' mathematical creativity in solving creative mathematical problem-situations, adequate assessment of students' creative behavior was of crucial importance. This was achieved in the following ways. First, the structure of creative mathematical problem-situations which invoke the functioning of the creative processes of sensitivity, redefinition and conjecturing were described explicitly in terms of the three parameters (domain, operation and range). Secondly, the three major characteristics of inventive divergent responses produced in solving problems of this creative structure were shown to be fluency, diversity and rarity of ideas. Thirdly, each problem of the three creative tests (CG1, CMG and CG2) was shown to be an appropriate and relevant mathematical situation reflecting the functioning of the three creative processes. Finally, special scoring procedures were developed to score students' responses to these test items in a more objective and accurate way.

Two new concepts were postulated to characterize the inventive divergent products (responses) to creative mathematical problem-situations defined in our study. Besides the common notion of fluency, which indicates the ability to produce a large quantity of mathematically appropriate responses, diversity and rarity were

defined. Diversity refers to the ability of utilizing a variety of relevant and significant categories which involve the use of related mathematical concepts or creative processes. Special procedures were developed to establish a set of significant "Diversity Categories" for each creative problem of the three creative tests, prior to the actual scoring of students' responses. Rarity then indicates the ability to employ original and infrequent diversity categories in response to those creative situations. These scoring procedures were shown to be superior to the often subjective, ambiguous and sample-biased types of scoring procedures employed by many researchers.

All instruments used in our study were found to be valid. Item data on GR7, A5T and MAR were not available for computation of reliability coefficients. On all other measures except one, reliabilities were high enough to establish the consistency of these measures. The reliability of CG1, though less satisfactory, was deemed adequate for the purposes of the present study.

Evaluation of the Inventive Method

The Nonequivalent Control Group Design was employed to examine the effectiveness of the Inventive Method (IM) of instruction in terms of students' terminal performance. The control group was taught by the Traditional Method (TM) of instruction. Analyses of students' entrance behavior measured in terms of the 5 pretests (GR7, A5T, MAR, SEC and CG1) indicated that the students of

the two groups were comparable with respect to their mathematical background, Euclidean level of geometric maturity and mathematical creativity. Information and data obtained on classroom teaching-learning activities indicated satisfactory implementation of the two instructional methods (IM and TM) as planned. These results provided valid grounds for further analyses of students' performance on our four posttests to determine the merits or demerits of the IM.

Two types of measuring instruments were deemed important for assessing the outcomes of the IM: (1) inventive divergent and (2) traditional convergent.

(1) Based upon Boychuk's creative problem solving model, the IM was designed to stimulate and enhance students' mathematical creativity. Two creative tests were therefore constructed to assess the treatment effects on students' ability to tackle creative mathematical problem-situations. Since students of the IM group were exposed to creative experience in the area of motion geometry, the Creative Motion Geometry Test (CMG) was devised to measure students' ability to react to the inventive divergent type of motion geometry problems creatively. To further determine whether the IM also promoted transfer of learned creative problem solving processes to novel geometrical situations, the Creative Geometry Test II (CG2) was constructed. The IM group out-performed the TM group on both CMG and CG2. Given the well-established validity and reliability of both measures, it is safe to conclude that the IM was effective in achieving our objectives of (a) promoting students' mathematical creativity in solving content-related mathematical

problem-situations, and (b) facilitating the utilization (transfer) of creative processes learned to more general mathematical situations.

(2) Apart from these explicit objectives of fostering creative thinking, it was also deemed important to determine whether the IM treatment did affect the quality of mathematics learning judging by "conventional criterion", i.e. students' performance on traditional convergent type of mathematical problems. For this purpose, the Traditional Motion Geometry Test (TMG) and the Transfer Area Test (TAR) were employed. TMG was designed by the school board to assess students' understanding and mastery of basic concepts of motion geometry, while TAR was specially constructed by us to assess students' ability to transfer learned concepts of geometric transformation to traditional types of geometrical problem-situations. The results showed that the IM group attained the same level of achievement on TMG as that of the TM group, but performed better than the TM group on TAR. Again, the well-grounded validity and reliability of TMG and TAR enabled us to conclude that mathematical knowledge of the kind deemed important by conventional criterion, -- i.e. mastery of prescribed content knowledge -- can be effectively taught through the IM. Furthermore, the IM shows itself capable of facilitating the utilization of mathematical concepts learned more effectively in novel mathematical situations.

DISCUSSION AND CONCLUSIONS

> It appears that the best way to insure a population of knowledgeable individuals is not by direct instruction in knowledge but by education for skills related to curiosity, problem-seeking and -solving, creative production and expression, and self-initiated learning (Cole, 1972: 36).

In our opinion, these should be the objectives of mathematics education, since mathematical activity is essentially creative problem solving. The present study attempted to achieve some of these objectives through a curriculum implementation of creative problem solving in junior high school mathematics. In planning our study, we were confronted by the problem of lack of "theoretical unity" in creativity research (Treffinger, Renzulli and Feldhusen, 1971: 107). A first necessary step was therefore to justify the theoretical foundation underpinning development of the curriculum and instructional components. Careful analysis of relevant literature led us to adapt Boychuk's creative problem solving model to the needs of our study. This model was operationalized to guide subsequent development of the Inventive Method of instruction as well as the evaluation instruments.

The achievement and results of the present study have, we feel, some significant implications for further research as well as school mathematics teaching. These results and their implications will be discussed in the following sections vis-a-vis six dimensions: (1) the creative problem solving model, (2) operationalization of theoretical model, (3) assessment of creative

thinking, (4) the Inventive Method of instruction, (5) effects of the Inventive Method, and (6) creative thinking and curriculum implementation.

Creative Problem Solving Model

We identified a four-process creative problem solving model, which had been validated by Boychuk (1974), as the theoretical foundation of our investigation. Though only the first three processes of sensitivity, redefinition and conjecturing were eventually adopted, this Three-Process Model defined, also basically includes the "hypothesis-construction" phase of the fourth process, verification. Boychuk showed that students' creative behavior in mathematical situations can be analysed in terms of these four processes, whose existence was substantiated by appropriate factor analytical methods. Clearly, a further step that might be taken to strengthen the validity of Boychuk's model is to demonstrate that an instructional method can be devised to influence the functioning of these unobservable processes. Our study was based on a three-process model which includes the hypothesis-construction dimension of the fourth process of verification. Valid measuring instruments were created, objective and well-grounded scoring procedures developed, and reliable data obtained. The results showed that students' mathematical creativity defined by our three-process model was effectively enhanced by the specially designed Inventive Method of instruction. To some extent, therefore, the outcomes of

the present study confirmed the validity of Boychuk's creative problem solving model.

Though Boychuk's model was shown to be an adequate theoretical framework for our study, it is acknowledged, of course, that creative problem solving in mathematical situations can be conceptualized within some other frameworks. One such conceptualization is Polya's notion of "heuristics". Boychuk had substantiated her theoretical model by reference to relevant problem solving processes in Polya's heuristic method, such as "guessing", "decomposing" and "recombining". Polya's heuristics, however, is meant to be prescriptive rather than descriptive, "heuristics" being "the study of the ways and means of discovery and invention" (Polya, 1966: 128). The general heuristic problem solving procedures of understanding the problem, devising a plan, carrying out the plan and looking back are supposed to constitute an effective and efficient way of solving problems. According to Polya (1973), generalization, specialization and analogy are important mental operations in mathematical problem solving. Generalization is reasoning from a given set of objects to a larger set containing the given one. Specialization is reasoning from a given set of objects to that of a smaller set contained in the given one, while analogy is reasoning based on similarity between sets of objects. Work on computer simulation of human behavior has produced evidence for the validity of Polya's observations on the problem solving process (Kilpatrick, 1969: 163; Higgins, 1973: 179). Since the

processes of generalization, specialization and analogy appear to be helpful in generating divergent solutions to creative problem-situations, it may well be possible to construct a creative problem solving model that incorporates Boychuk's creative processes as well as Polya's heuristic processes. Certainly, this points to one interesting direction further research could take.

Operationalization of the Theoretical Model

Detailed logical analyses of the structure of creative problem-situations demonstrated that no exclusive independent measure can be obtained for each of the three processes from students' responses to any such situation. This conclusion was confirmed by Boychuk's (1974: 219) empirical data. Furthermore, fluency, diversity and rarity of ideas were shown to be three major indispensable characteristics of inventive divergent responses to any creative problem-situation reflecting the functioning of the creative processes of sensitivity, redefinition and conjecturing. For the purposes of the present study, mathematical creativity was therefore defined operationally in terms of fluency, diversity and rarity of ideas. To clarify the logical connection between the creative processes and the latter three characteristics, we focused on the structure of creative problem-situations. The functioning of the three creative processes can only be inferred from students' responses to problem-situations which provoke or necessitate the utilization of the processes. We therefore began by describing

clearly the structure of such inventive divergent types of problem-situations in terms of the three parameters (domain, operation and range) identified. The usage of this three-parameter language was justified by reference to literature on "rule learning". In this way, the responses to such problem-situations could be regarded as manifestations of the functioning of the three creative processes. For the purpose of analysis, we quantified these creative responses on the three dimensions of fluency, diversity and rarity. In short, by showing explicitly the above relations between the three creative processes and the three basic characteristics of creative responses, which also constitute the criteria of measurement, we essentially resolved satisfactorily the "discrepancy issue" for our study.

A clear distinction was made between the conceptual definition consisting of the three creative processes of sensitivity, redefinition and conjecturing, and the operational definition stated in terms of fluency, diversity and rarity. The congruence between our conceptual and operational definitions of mathematical creativity was established via the structure of creative mathematical problem-situations. In contrast, many theorists define creativity in terms of such notions as fluency, flexibility, originality, transformation and elaboration, and then proceed to construct instruments which supposedly measure some or all of these "constructs" (processes or operations) directly. Furthermore, they seem to assume that "creativity" is not identical to fluency, flexibility, originality, transformation and

elaboration, which serve only as indices of creativity. In effect, they define creativity conceptually in terms of those constructs, and then regard these constructs as operational definition of the more complex notion of "creativity". To justify this dual usage of the constructs, congruence between the "broad" conceptual use of these notions and the "narrow" operational use of the same notions should be established by showing that these notions (constructs) are necessary and sufficient for any subject of the defined population to tackle adequately the creative tasks of those measuring instruments. Unfortunately, such attempts at justification have apparently not been undertaken by the theorists concerned. As Treffinger, Renzulli and Feldhusen (1971: 109) pointed out, creative tasks which "on the face" appear to be attractive measures of postulated "creativity" may in fact reflect quite different abilities (constructs).

By analysing the structure of creative problem-situations, we resolved this "discrepancy" between conceptual and operational definitions of creativity, and also avoided some of the difficulties encountered by Boychuk (1974: 219) in constructing creative problems. The present study showed fluency, diversity and rarity to be three necessary characteristics of divergent products (responses) to our inventive divergent type of mathematical problem-situations. No claim, however, is made that these characteristics are sufficient to enable us to equate sensitivity, redefinition and conjecturing with fluency, diversity and rarity. In other words, creative problem solving in mathematical situations reflecting the

functioning of the three creative processes may well exhibit other characteristics besides fluency, diversity and rarity. Some plausible characteristics, such as complexity, condensation and elaboration, appear promising from our observation of students' responses in the present study, as well as results of other studies.

Within the framework of our scoring procedures, "diversity of ideas" indicates the ability to utilize many distinct "diversity categories". No distinction, however, was made to discriminate between, say a student who employs 10 such categories to produce 10 different responses, from the student who employs the same 10 categories to produce a single response. In fact, the former would be regarded as more creative than the latter due to higher fluency scores. We would, however, then be penalizing the latter student who appears to be extremely creative, since high ingenuity is certainly required to put 10 diverse categories together to create a coherent and complicated solution. In the present study, there were a few such students who did not produce many of those mathematically appropriate but obvious solutions, but whose small number of solutions were of a rather complicated nature. Boychuk (1974: 135) also notices this characteristic of "complexity" manifested in her sample of students.

Condensation was first suggested by Jackson and Messick (1965) to indicate the degree to which a creative response manifests a unified and coherent relationship between simplicity and complexity. Since the notion of "complexity" seems to imply "condensation", can we then subsume condensation under complexity?

Is it feasible and promising to define these two notions in some way so as to make a logical distinction between the two? These would seem to be meaningful questions to pursue in further research.

Elaboration is one of Guilford's mental operations, indicating the ability to state many details related to the creative response. Again, is "elaboration" part of "complexity"? Can we distinguish "elaboration" from "complexity" and "condensation"? In our opinion, all these questions should be discussed on the basis of the structure of creative problem-situations. It is probably too ambitious to attempt to incorporate all these characteristics of fluency, diversity, rarity, complexity, condensation and elaboration and others into one type of problem structure. On the other hand, investigation of a few types of problem structures, which optimally reflect the functioning of the three creative processes, but result in various types of creative responses (products) that optimally manifest different sets of these characteristics, would seem more likely to yield promising results.

Assessment of Creative Thinking

One of the important tasks of the present study was to evaluate the effectiveness of the Inventive Method (IM) of instruction developed from Boychuk's theoretical model. The treatment effects of the IM on students' ability in solving creative mathematical problem-situations were therefore our primary concern. Students' behavior in such creative problem-situations was measured

in terms of the responses (solutions) they generated in solving these problems. In order to argue that students' divergent responses to the creative problems of the three constructed creative tests (CG1, CMG and CG2) were manifestations of the functioning of the creative processes of sensitivity, redefinition and conjecturing, we showed that all of these problems were valid creative situations necessarily invoking the use of these creative processes. The employment of the criteria of fluency, diversity and rarity to score students' creative responses were justified by demonstrating the logical relation between the creative processes and these criteria which also constituted the operational definition of creativity for our study. Special scoring procedures were further developed to obtain objective, accurate and reliable measures according to our three-fold criteria. In this way, we managed to provide a reasonable response to the common criticism of creativity research that the "link" between performance on creative tests and the multiplicity of ways in which creative processes function is dubious (Entwistle and Nisbet, 1972: 168). An adequate foundation was also laid for drawing valid conclusions from students' terminal performance on our creative tests.

The special scoring procedures, which required the establishment of a set of significant diversity categories for each creative problem prior to the scoring of students' responses, turned out to be extremely useful for assessing creative thinking. In view of the complexity and subjectivity of scoring divergent responses to

"open-ended" creative problem-situations, such procedures of setting up scoring criteria <u>independent</u> of the responses will, we argued, yield more objective and accurate measures of diversity (flexibility) and rarity (originality) of ideas which are necessary characteristics of creative thinking. The process of identifying these diversity categories was essentially a content-analysis of each creative problem-situation within the conceptual framework of the present study. As a certain degree of subjective judgement was involved, the sets of diversity categories cannot be treated as final absolute categories and are certainly open to discussion. To facilitate such discussion, as well as for the following reasons, documentation was provided outlining the basis for the final establishment of all the six sets of diversity categories together with the actual sets employed for scoring. First, this will set the ground for further discussion vis-a-vis the validity and adequacy of our scoring procedures as well as the results obtained in the present study. Secondly, our documentation will also facilitate the effective use of our constructed instruments for measurement of mathematical creativity. Thirdly, this will permit further replication of our study to validate the results we obtained.

It must be noted, however, that these sets of diversity categories established are not meant to be final for the creative problems employed. This qualification stems from the fact that it is still relevant to ask whether these sets of categories developed remain invariant over larger and many samples of students? Specifically, we wonder whether for each of our problems, such a

set of categories can be standardized as a "norm" for a specific problem. Of the six creative problems constructed, only the first problem of CG2 was used in both pilot and main studies. In this case, the diversity categories developed for the pilot study were adopted in their entirety without modification for the final experiment. This provides a small and exploratory piece of evidence for affirmative answers to the foregoing questions. Clearly, large-scale studies can and should be conducted to establish standardized "diversity categories" for each of our creative problems. At the same time, of course, similar norms for the rarity score could also be established.

These suggestions for further research are consistent with the basic assumption of most of the research studies that attempt to measure creative thinking, namely, that creativity, like any other human mental ability, is normally distributed in the population. If further studies converge to some "norm" for each creative problem, it would not only substantiate our assumption, but also facilitate communication of research findings.

Inventive Method of Instruction

It appears that a creative teaching strategy can be developed along two different dimensions. First, following Polya's heuristics for problem solving in general, one can operationalize the three creative processes by identifying certain "creative heuristics" for solving creative mathematical problems. We had

attempted, in our pilot study, to discuss a typical process-type of creative problem-situation with students so that experience and ability could be developed to enable them to transfer the learned "creative heuristics" to related novel situations. The effects of such types of instruction were not observed. Though during classroom discussion, students were prompted to solve problems by utilizing some of the creative processes, the functioning of these processes in a given situation seemed to have certain "problem-specific" features. In short, solving problems using "creative heuristics" failed to promote creative problem solving ability. It is rather interesting to observe that a recent study by Post and Brennan (1976: 64) suggests that the experience of tenth-grade students in solving geometrical problems using parts of Polya's heuristic processes is "not effective in promoting increased problem-solving ability." Nevertheless, in our opinion, it still seems worthwhile to attempt to formalize "creative heuristics" that would enable students to respond effectively to creative mathematical problem-situations.

The other alternative is to expose students to creative mathematical situations that provoke the functioning of the three creative processes, but emphasize the divergent production of ideas to these situations. We have logically argued that, (a) mathematically appropriate solutions to these situations implied to some degree the utilization of these creative processes, (b) encouraging divergent production of solutions would be likely

to facilitate the functioning of these processes, and (c) the resultant creative responses could be conveniently measured on the three characteristics of fluency, diversity and rarity. This approach was shown to be promising by the results of our pilot study. The focus of our Inventive Method (IM) of instruction therefore was to "encourage inventive divergent production." Certain features of the IM, which underpin our views on what constitutes effective instruction for promoting mathematical creativity in school learning, should now be discussed.

First, the "display function" of the IM obviously appeared at first sight to contradict the objective of creative teaching, since a highly structured and directed Development of Concepts (DC) phase was adopted to display the content of motion geometry to ensure adequate mastery of subject matter. This particular tactic is justified, however, by reference to instructional research and the results of some creativity studies in school mathematics (e.g. "discovery method" and "mathematizing mode"), all of which affirm the necessity of prerequisite knowledge for both inventive divergent and traditional convergent thinking. In our opinion, meaningful mathematical learning is contingent upon the accessibility of relevant mathematical content knowledge and mathematical processes (skills). Since the present study is the first attempt to implement Boychuk's theoretical model at the classroom level, this "content" dimension was kept constant for both the experimental and the control groups, for the purpose of effective experimental control, as well as the apparent

effectiveness and efficiency of expository method in imparting content knowledge to students.

Secondly, the IM appeared to be highly structured. Both the IM and the TM followed the same sequence: (a) development of concepts (DC) through direct expository method, (b) assignment of a set of exercises (IE or TE) as homework, and (c) class discussion of previously assigned exercises, (ID or TD). The instructional formats were quite similar. The differences occurred in the content of the exercises and discussion. For the 19 sets of IE, 60% were of the divergent type of problem-situation which encourage students to search for a variety of appropriate solutions to a given situation. Since these problems were structured to stimulate the functioning of the three creative processes, solving these IE's provoked the utilization of these processes. The fact that all the students of the IM group attempted most of the IE further indicated that students of the IM group were engaged in creative problem solving activities. With respect to the discussion phase of the two instructional methods, the Observer Rating Scale confirmed that the teacher tended to be more flexible and exhibited a more "inventive approach" during the ID phase of instruction (IM), in comparison with the TD phase of the TM. Furthermore, classroom observations undertaken by the investigator indicated that the teacher did follow the prescribed procedures in implementing the two instructional methods. Most of the suggested solutions of the IE were discussed along the lines of the implied creative processes.

In short, the IE and ID ensured that inventive divergent thinking utilizing the three creative processes were encouraged and rewarded.

Thirdly, there was built-in reinforcement that provided the needed environmental and motivational components of effective learning. Besides engaging students in inventive divergent thinking through the IE and the ID, the IM also took into account some variables of the "affective domain". The sequential structure of the DC, IE and ID phases of instruction, with emphasis on inventive divergent production, helped to establish in students' minds the expectation that divergent production was required and would be rewarded in problem-situations that asked for a multitude of solutions. The IM also fostered a more flexible and open attitude towards creative problem-situations.

Finally, the IM focused on the teacher's role of displaying the content to be learned, and of controlling the learning activities of the students. We did not take into account the students' role in the teaching-learning process. Furthermore, we only monitored the teacher's action during the classroom observation. However, this was not to belittle students' contributions towards effective learning. It was simply deemed appropriate, at this stage of research based on Boychuk's model, to facilitate better experimental control by concentrating on the "teacher component" of instruction. It also appeared to us that students' effects on the quality of instruction, with respect to the "inventiveness" of teaching, was indirectly reflected on the Observer Rating Scale, which mainly quantified the interaction between the teacher and students. The

investigator noticed, however, some aspects of students' classroom behavior which might have significant implications for further research.

During the first two discussions (ID) of the assigned sets of IE, most students of the IM group appeared quite reluctant to suggest alternative solutions to a problem where a correct solution was found. A few students seemed rather impatient and one of them even asked, "isn't one answer good enough?" The teacher had to emphasize the requirement of the problem several times to overcome this sort of "mental inertia" and to stimulate the production of divergent solutions. After the first week, however, many students reacted quite spontaneously to the teacher's questions, hints or prompting. Some interaction between students did occur, where students argued among themselves about the correct solutions, or suggested improvements and alternatives to other's solutions. But owing to the constraints of time, the teacher had to terminate many of these interesting discussions to go on to the next problem.

When the IM group was discussing the solutions to the third problem of Lesson No. 8, where students were required to find the number of diagonals of polygons with different numbers of sides, from 3 to 10, one student suggested the following "way" for computing the number of diagonals of any n-sided polygon:

> You take the number of sides and multiply this number by itself. Subtract from this the number of sides. Then divide by 2. Take away the number of sides again.

This is in fact a "verbal" version of the correct formula for an

n-sided polygon:

$$\text{Number of Diagonals} = \frac{n \times n - n}{2} - n$$
$$= \frac{n(n-3)}{2}.$$

Unfortunately, when the teacher tried to verify this verbal formula by asking the student the number of diagonals of an octagon, she gave the incorrect solution of 28, probably because she forgot to "take away the number of sides". The teacher nevertheless praised the student for her effort and proceeded to the next problem.

This episode was not meant to show the effect of the Inventive Method of instruction. However, like the interaction between students mentioned above, it does raise the question of what role students actually play in an inventive approach of teaching. Is this role passive or active? Most importantly, can students' reactions during the teaching-learning process and the effects of the IM on the quality of students' interaction be measured through some instrument similar to the Observer Rating Scale? Obviously, attempts to locate answers to these questions would need to incorporate the "student" component into the whole conceptual framework of research.

Furthermore, mathematical creativity has been found to correlate significantly with intelligence (Prouse, 1965) and other cognitive factors, such as verbal ability (Boychuk, 1975). It seems plausible to suspect therefore that the IM might have interacted with students' cognitive abilities. Are students benefiting equally from the IM regardless of their mental abilities, or is the impact

differentially distributed according to different levels of such abilities? We should also note that attitude towards mathematics has been shown to correlate significantly with mathematical creativity (Evans, 1964), while motivating conditions seem to affect creative productions (Elkind, Deblinger and Adler, 1970). Would such affective factors interact with the IM? These are clearly relevant questions to ask, as their answers will help deepen our understanding of the potential effects of the IM in school mathematics. We therefore suggest further research to explore the interaction effects of the IM with various relevant cognitive and affective factors in mathematics learning.

Effects of the Inventive Method

We should not be surprised to find that both IM and TM groups achieved the same level of performance on the Traditional Motion Geometry Test (TMG). A careful scrutiny of the 40 items of TMG (see Appendix E) shows that these are mainly "immediate recall" type of content items. Since the two groups were exposed to equivalent types of expository instruction on basic concepts of motion geometry during the DC phase of instruction, and the two groups were comparable in all relevant aspects identified for our study, we would expect the same level of achievement for both groups on TMG.

With respect to the two creative tests (CMG and CG2), the Inventive group (IM) was shown to have out-performed the Traditional

group (TM). A special feature is evident from the profiles of the two groups (Figure 7.3). For both tests, the differences between the respective fluency and diversity scores of the two groups are rather narrow, while the rarity scores of the two groups differ remarkably. This phenomenon springs from the inherent nature of our scoring procedures. The fluency score indicates the number of distinct and appropriate responses. The production of <u>many</u> <u>distinct</u> responses to a creative mathematical problem-situation tends to utilize many different mathematical concepts or creative categories deemed significant for that problem. This results in turn, in high diversity scores and eventually higher rarity scores. Since the diversity scores are the total number of distinct diversity categories, whereas the rarity scores are the weighted sum of the diversity scores, where the weights could be 0, 1, 2, 3 or 4, the difference between the diversity scores of the two groups is likely to be magnified by the weights, and consequently accounts for the wide difference between the rarity scores. To some extent, this further demonstrated the logical consistency of our scoring procedures.

The superior performance of the IM group over that of the TM group on CMG and CG2 showed that the Inventive Method of instruction did enhance students' mathematical creativity. Since the primary purpose of CG2 was to assess students' ability to transfer the creative problem solving processes learned under the IM to novel geometrical situations, the results verified the occurrence of such transfer of learning. In addition, the content-

analyses of CG2 pointed to the possible utilization of concepts of geometric transformation to solve the two creative problems. Given that the IM basically exposed students to creative problem solving <u>in</u> motion geometry and our scoring procedures did include a dimension to take into account the utilization of geometric transformations, these results also indicate positive interaction effects between the IM and the content of motion geometry. In short, the Inventive Method of instruction in school mathematics not only enabled students to solve mathematical problems creatively, but also promoted the creative utilization of specific mathematical knowledge learned in other novel mathematical situations.

Students of the IM group also performed better than their counterparts in the TM group on TAR. The question of "what has been transfered" needs further examination. The original purpose of TAR was to provide problem-situations to facilitate the transfer of content knowledge of motion geometry to area-finding situations. There was, however, only one brilliant student of the TM group, who applied geometric transformation to TAR and thus scored the maximum of 40 points. In contrast, all students of the IM group employed geometric transformations to solve some or all the 12 non-standard problems of TAR. We might conclude that the concepts of geometric transformation learned through the IM had been successfully transferred to solving problem-situations of TAR. Further examination of our 19 sets of IE revealed that it might well be the effects of the two particular problems of the IE. The last problems in the first two lessons are creative problem-

situations prompting students to transform one figure into another by performing appropriate motion of certain parts of the former figure. Though these two problems did not ask for areas, they did provide students with the important principle needed to solve the problems of TAR. The possible effects of these two problems were anticipated during the development of the 19 IE materials. They were not eliminated, however, because of their provision of rich problem-situations nicely reflecting the functioning of the three creative processes. Though the effects of the IM treatment appeared to be confounded with the possible effects of these two exercises, we would still suspect significant treatment effects on TAR on the basis of the significant treatment effects on both CMG and CG2.

In summary, the Inventive Method (IM) of instruction was effective in enhancing students' mathematical creativity while at the same time enabling students to attain a level of achievement on basic mathematical content equivalent to that of students under the traditional expository method of instruction. The IM further enabled students to tackle novel geometrical problem-situations creatively, and facilitated the utilization of learned materials by students to other conventional types of geometrical situations.

Finally, a short comment on the creative tests constructed for our investigation is probably helpful for future replication of the present study. Due to time constraints, each of our creative tests (CG1, CMG and CG2) consists of only two problem-situations. Besides, asking students to respond to these

creative problems under strict examination arrangements inevitably introduced a "speed" factor into these measures. It is generally accepted, of course, that many creative works of science and art are usually not produced under strained circumstances. Further research along the lines of our study should therefore also increase the number of test items, alter the format of the problems, devise more natural and relaxed testing situations, or measure students' creativity by grading samples of their homework. Studies aimed at developing better measuring instruments are valuable, since these instruments would enable us to assess more accurately the effectiveness of the Inventive Method of Instruction.

Creative Thinking and Curriculum Implementation

A basic motivation underlying our undertaking this piece of research has been that fostering creative thinking is of primary importance in school mathematics. Much of contemporary research in mathematical creativity generally focuses on the building of a theoretical model of creativity or the assessment of creative thinking. Few researchers, however, specifically explore the feasibility and effectiveness in applying these models to natural classroom settings. The present study attempted to operationalize one such model, and show how creative thinking can be stimulated and enhanced in school mathematics. The primary objective of the study was therefore to develop, on the basis of a valid model of mathematical creativity, an effective instructional method with relevant instructional materials.

The results of our endeavor showed that through appropriate operationalization of a well-chosen model of creative problem solving, a creative method of instruction could be developed and successfully implemented, in a significant content area of junior high school mathematics curriculum, to promote creative thinking in mathematical problem-situations. More importantly, besides showing that creative teaching does enhance creative thinking, the present study also demonstrated that creative instruction can accomplish what conventional teaching is supposed to achieve -- i.e. the learning of specific mathematical content by students. Indeed, creative teaching can better facilitate the utilization of knowledge learned in this specific area of mathematics to novel mathematical situations.

Finally, the results of the present study showed in effect that if creative thinking is a desirable outcome of school mathematics curriculum, then this end can be attained effectively through classroom instruction designed in a certain way. The sequential approach adopted for the development of the Inventive Method of instruction with its various phases, thus provides a valid and meaningful basis for future attempts to formulate a theory of creative instruction.

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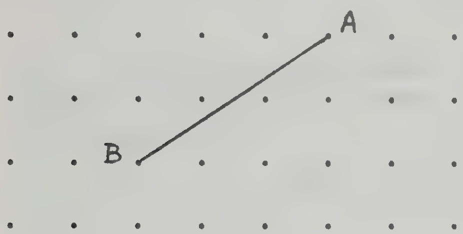
APPENDICES

APPENDIX A

INVENTIVE EXERCISES (IE)

AND SUGGESTED SOLUTIONS

I. Copy segment $\overline{AB}$ on your dot paper. Slide AB (4R,1D) and draw image $\overline{A'B'}$. Draw slide arrow joining A to A' and B to B'.



1) What is the image of:

point A ______

point B ______

$\overline{A\ B}$ ______

2) Write down all the relations you notice about a slide (as many as possible).

______

______

______

______

3) Can you test for congruence between $\overline{AB}$ and $\overline{A'B'}$ in more than one way? How?

______

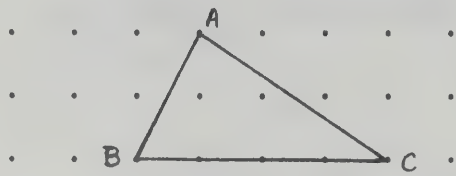
______

______

4) How do you know that $\overline{AB}$ and $\overline{A'B'}$ are parallel? ______

______

II. Copy $\triangle$ABC on your dot paper and draw the image for the slide (3R,2U). Label image $\triangle$A'B'C'.



1) What is the slide image of:

$\overline{A\ B}$ ______

$\overline{B\ C}$ ______

$\overline{C\ A}$ ______

2) Now join each point to its image and write down all the relations you notice about a slide of a figure. ______

______

______

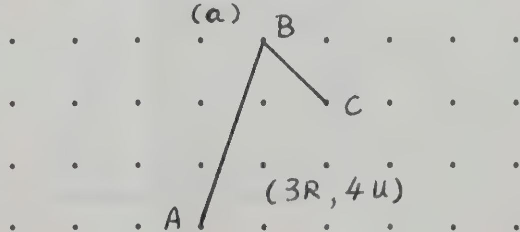
______

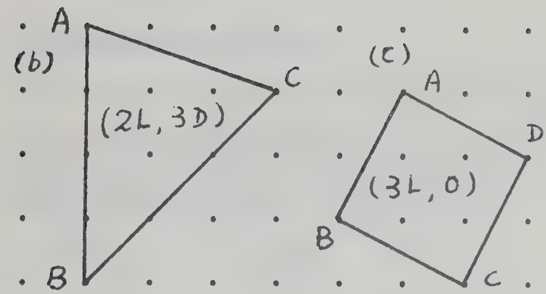
______

______

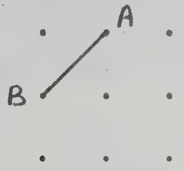
III. Copy each figure onto your dot paper and draw its image under the given slide. Join each named point and its image with dotted lines. Use the properties for slides, mark all congruent segments and indicate pairs of parallel lines.

(a)





IV. Suppose you are given the region consisting of 9 dots and the segment $\overline{AB}$.



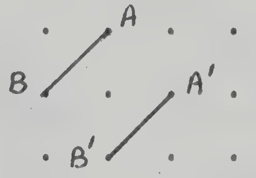
1) Make as many segments as you can with end-points on dots.

2) How many of these segments are congruent to $\overline{AB}$?

3) How many of these congruent segments are images of $\overline{AB}$ under a slide only?

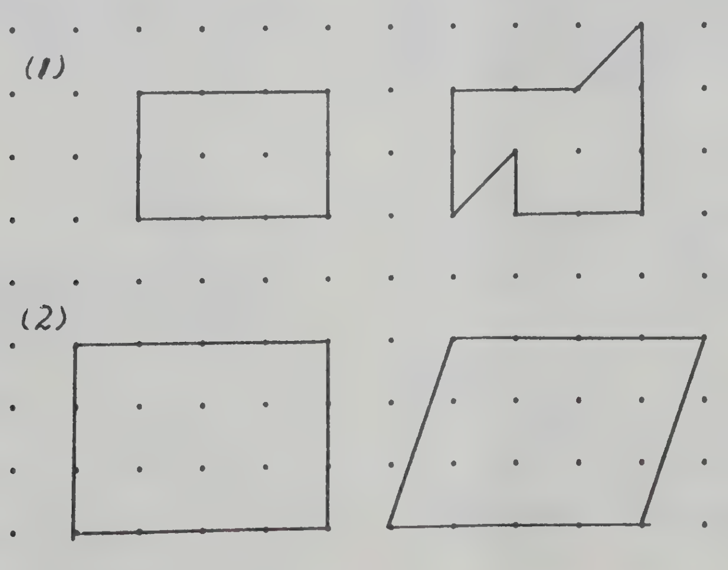
V. Suppose you are given the segment $\overline{AB}$ with its slide image $\overline{A'B'}$ within a closed region with 12 dots. Without sliding $\overline{AB}$ outside the 12 dots, in how many ways can you slide $\overline{AB}$ onto $\overline{A'B'}$?

Give all your solutions in the standard slide notation, i.e. (__R/L,__U/D).

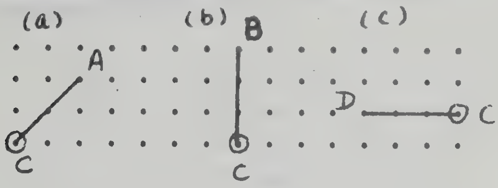


(Hint: 1. Can you use more than 1 slide?
2. What is the maximum number of slides you can use without repetition?)

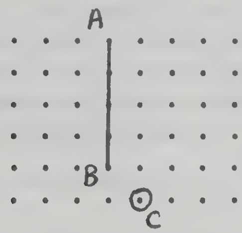
VI. Show how you can convince your friend that the following pairs of figures have the same area? (Hint: can you transform one into the other by a slide?)



I. Copy each figure on your dot paper and draw its ½-turn image. Mark the congruent segments.



II. Copy the figure on your dot paper and draw its ½-turn image. C is the turn centre. Label the image A'B'.

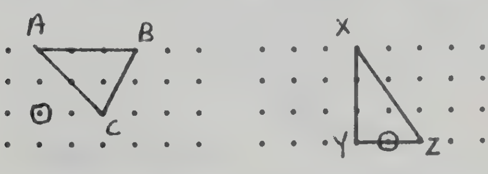


Join the points to their images and write down all the properties of ½-turn. ____________________

____

____

III. Copy △ABC and △XYZ onto your dot paper and draw the half-turn images. Label each image.



Join the points to their images with dotted lines and put down all the properties of a ½-turn.

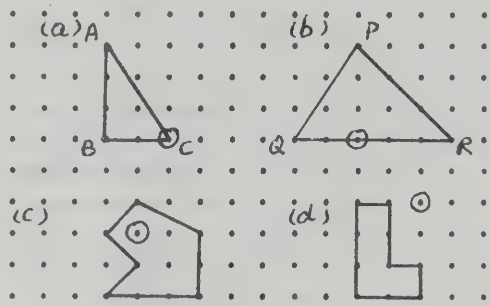
____

____

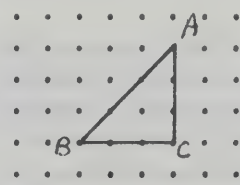
____

____

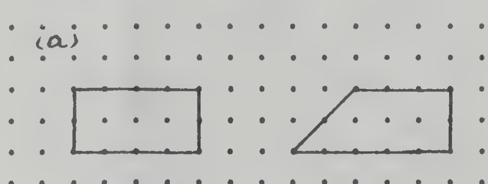
IV. Use various properties, draw the half-turn images on your dot paper without turning or tracing the original.

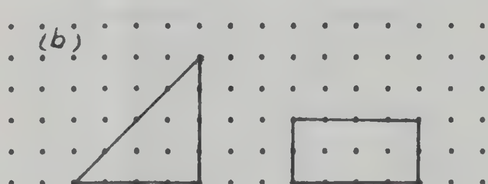


V. Copy △ABC onto your dot paper. Locate your own turn center, such that the ½-turn image of △ABC and the original △ABC will form a parallelogram.



VI. Show how you can convince your friend that the following pairs of figures have the same area? (Perform a ½-turn on some part of one figure.)

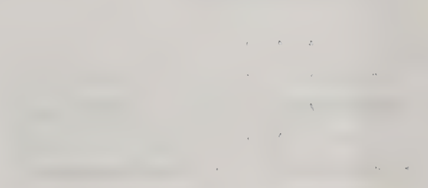


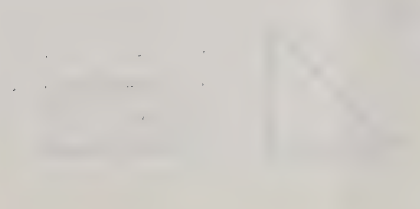


[illegible]

[illegible]

[illegible]



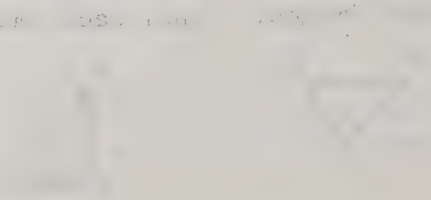


[illegible]

[illegible]

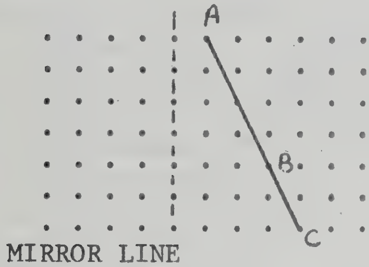
[illegible]

[illegible]



[illegible]

I. Copy the segment onto your dot paper and draw its reflection image. Label the image A'B'C'.



1) Join the points to their images with dotted lines and write down all the properties you notice about reflection.

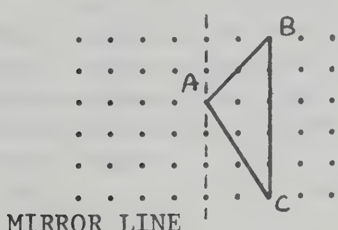
____

____

____

____

II. Copy the figure onto your dot paper and draw its reflection image, and join named points to their images.



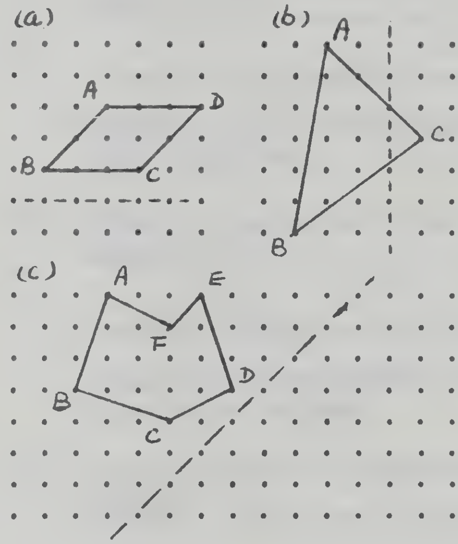
1) Put down all the properties of reflection you notice from this figure.

____

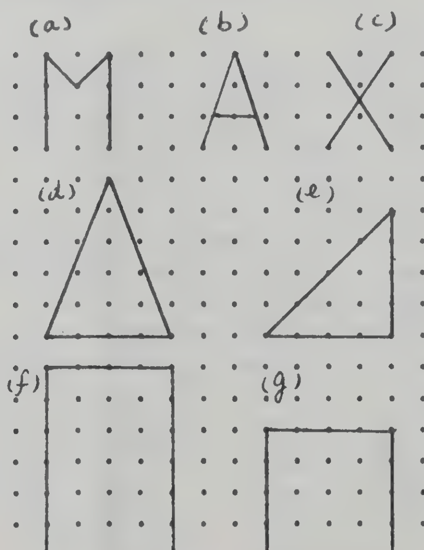
____

____

2) Using the properties you discovered, draw the reflection image of each of the following on your dot paper.

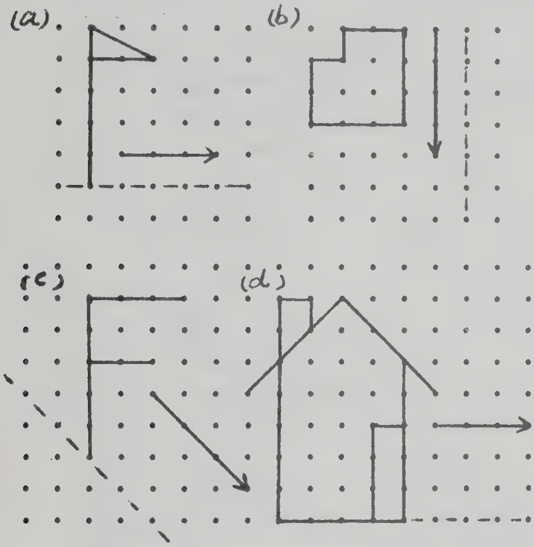


III. For each of the following, draw all the POSSIBLE positions of the mirror lines which would reflect the original figure onto itself, i.e. the reflection image would fall exactly onto the original figure. (Such mirror lines are called Lines of Symmetry.)



INVENTIVE EXERCISES LESSON NO. 4

I. Copy the following figures onto your dot paper and draw the images under the slide-reflections indicated.



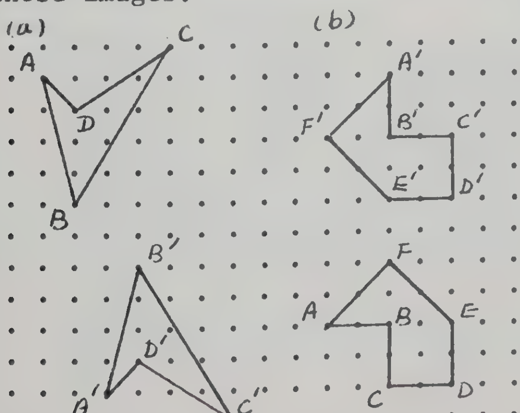
Write down all the properties of slide-reflection you notice.

____

____

____

II. Copy the following figures with their images onto your dot paper. Draw in the reflection lines and the slide arrows that would show the slide-reflections needed to obtain those images.



III. Figure A is the original. Some of the other figures are images of A and some are not.

1) Which of those are SLIDE images of A? ______________

____

Draw the slide arrows with the slide notations.

2) Which of those are ½-turn images of A? ______________

____

Locate the turn centers and draw in the arrows.

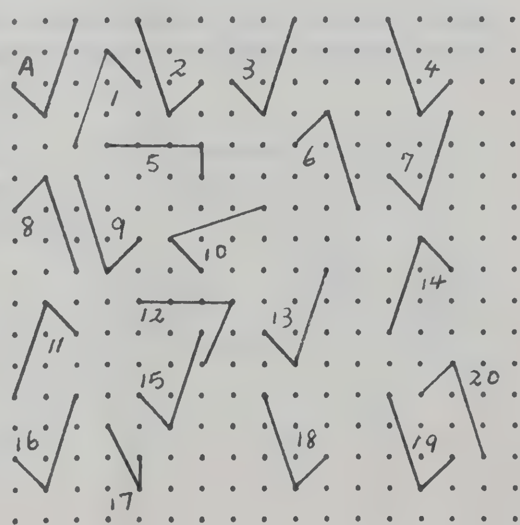
3) Which of those are REFLECTION images? ______________

____

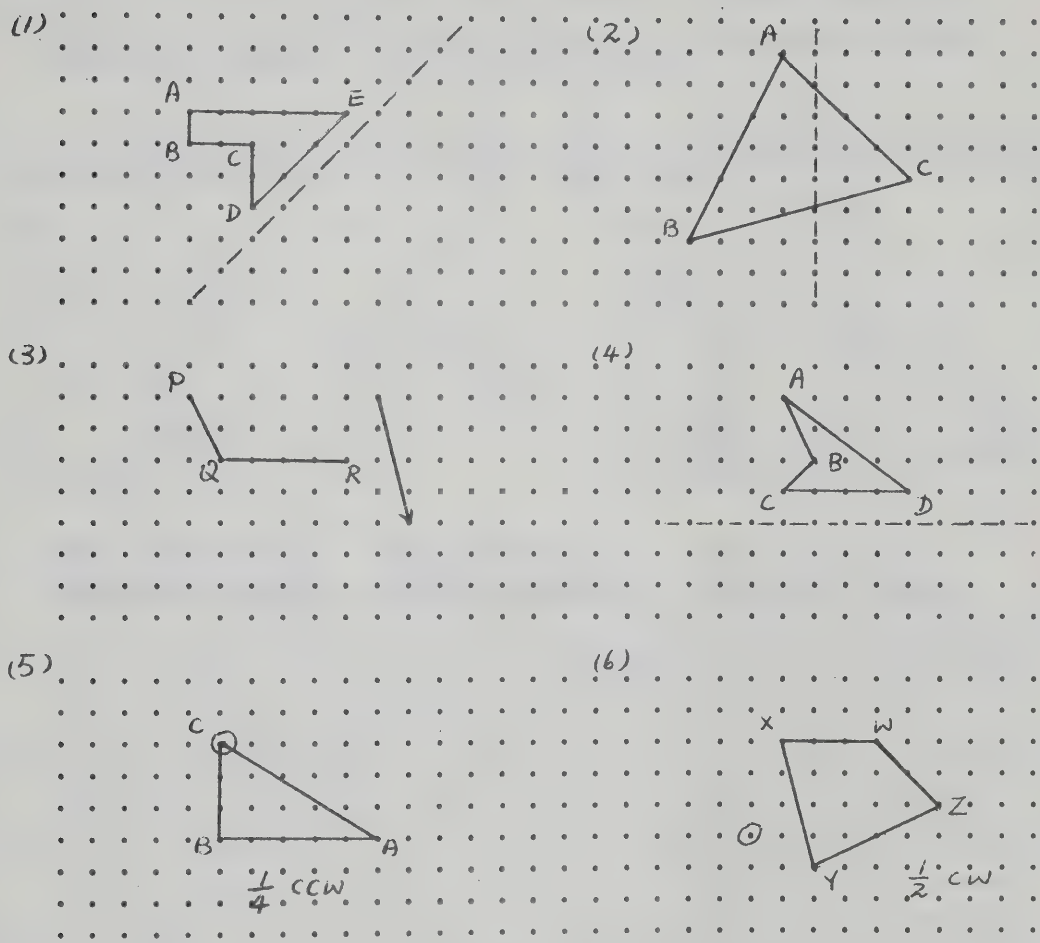
4) Which of those are images under SLIDE-REFLECTION?

____

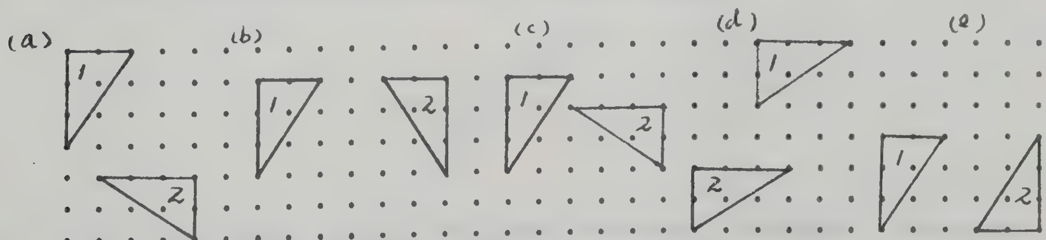
Draw in the reflection lines and the slide arrows.



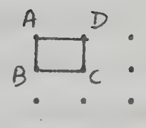
I. Draw the images of the given figures under the given glides:



II. Name and show the transformation which maps object 1 onto object 2 in each case below. Mark the correct symbols on the diagram, e.g. , or or .



III. You are given the square ABCD within a 9-dot region:

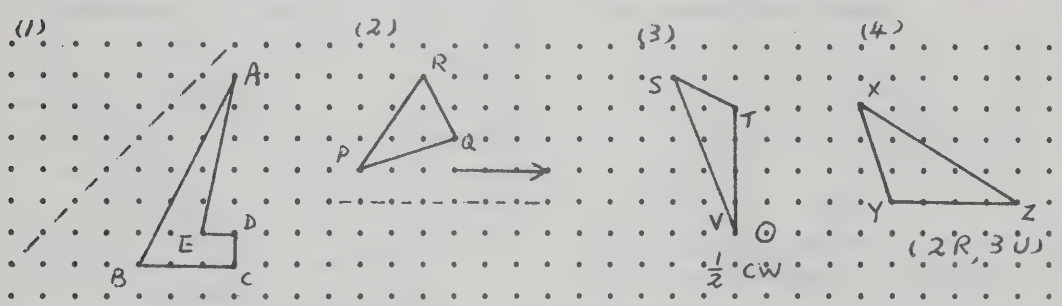


1) Using only those given dots as end-points, how many squares can you draw within this region? ________

2) How many of these squares are congruent to square ABCD? ________

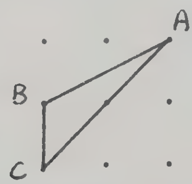
3) Describe the glide or glides required to obtain each of the squares in part (2) from the square ABCD.

IV. Draw the images of the figures under the given glides and label the image points, then fill in the chart below.



	Name the pairs of congruent segments	Name the pairs of parallel segments	Name the pairs of congruent angles
(1)			
(2)			
(3)			
(4)			

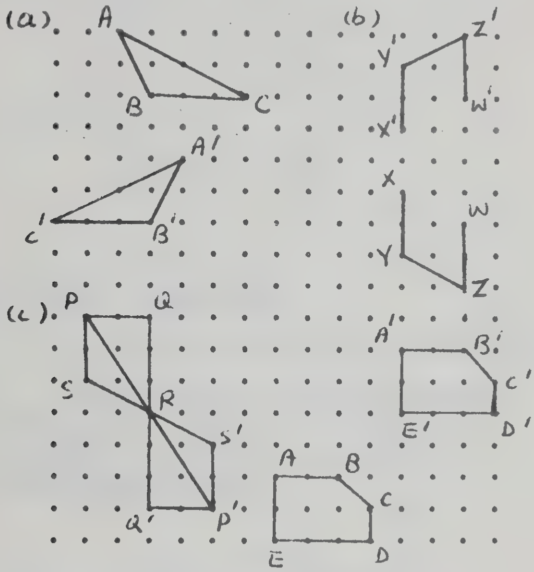
V. You are given a $\triangle$ABC within a 9-dot region.



(1) Use a new 9-dot region for each different triangle. Draw all the triangles that are congruent to $\triangle$ABC.

(2) Describe the glide or glides required to obtain each of the triangles in part (1) from $\triangle$ABC.

I. Using the following diagrams, answer the questions:



1) What motion produced

(a)__________ (b)__________

(c)__________ (d)__________

2) Complete the diagram with the appropriate symbols to indicate the type of glide, e.g.

$\odot$, $\longrightarrow$, ---------- .

3) Referring to the diagram, complete the correspondence:

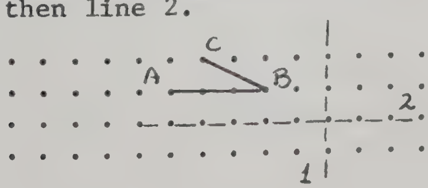
(a) [A,B,C] $\longrightarrow$ [A',B',C']

(b) [X,Y,Z,W] $\longrightarrow$ [ ]

(c) [P,Q,R,S] $\longrightarrow$

(d) [A,B,C,D,E] $\longrightarrow$

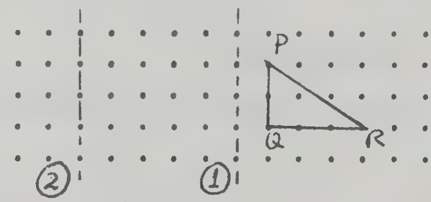
II. Copy ABC onto your dot paper. Draw the flip image about line 1 and then line 2.



What single motion is equivalent to these successive flips?

____

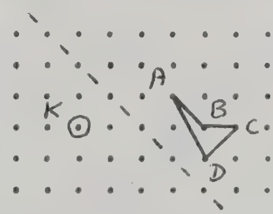
III. Copy $\triangle$PQR onto your dot paper and flip $\triangle$PQR about line 1, then about line 2, and draw the resulting image.



What single motion is equivalent to these successive flips?

____

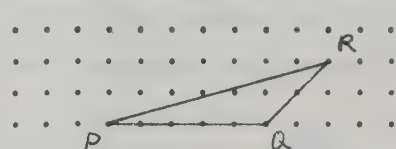
IV. On your dot paper find the image of ABCD by flipping and then making a half-turn about K.



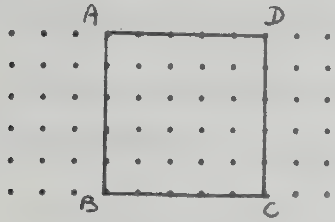
What single motion is equivalent to this combination of motions?

____

V. Using many different glides or combinations of glides form parallelograms with $\triangle$PQR.



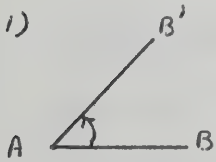
VI. Find all possible single glides that produce images of ABCD which fall exactly onto ABCD itself.



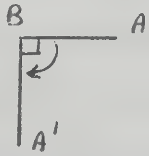
INVENTIVE EXERCISES LESSON NO. 6

I. Classify the following angles as to Acute, Right, Obtuse or Straight, by writing the name under each picture.

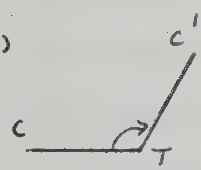
(1)



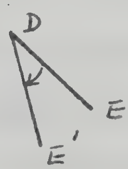
(2)



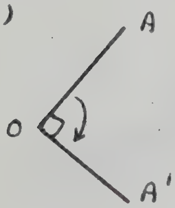
(3)



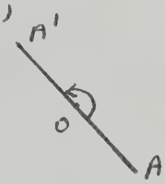
(4)



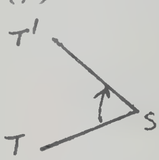
(5)



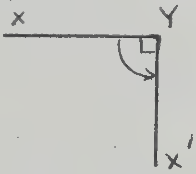
(6)



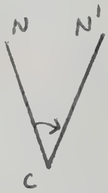
(7)



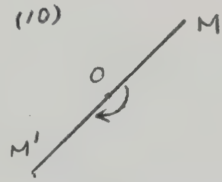
(8)



(9)



(10)



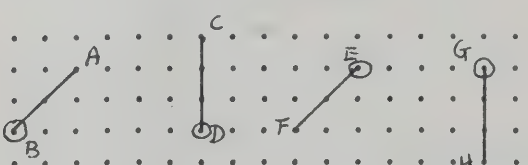
II. Copy the four segments onto your dot paper. Use the turn centres indicated and draw:

1) all acute angles with $\overline{AB}$;

2) all obtuse angles with $\overline{CD}$;

3) all right angles with $\overline{EF}$; and

4) all straight angles with $\overline{GH}$.



III. Write, in your own words, definitions of

1) right angle ______________

____

____

2) acute angle ______________

____

____

3) obtuse angle ______________

____

____

4) straight angle ______________

____

____

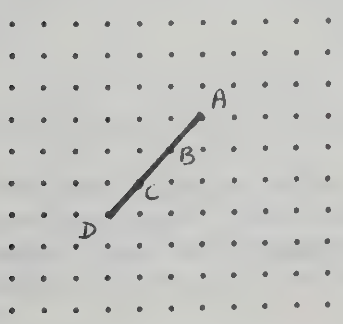
IV. Copy the segment ABCD onto your dot paper. Pick your own turn centres, and draw:

1) all possible acute angles;

2) all possible obtuse angles;

3) all possible right angles;

4) all possible straight angles.

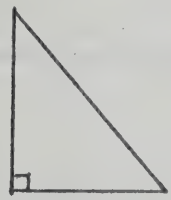


Do not draw all your angles on a single region.

Use a new region with segment ABCD for each different angle.

I. Classify each of the following triangles by their angles. Write your classification under each △.

(1)



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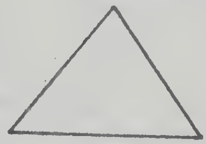
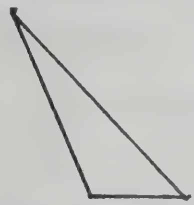
(2)



______

(3)



______

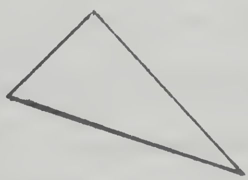
(4)

______

(5)

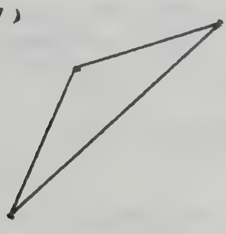
______

(6)



______

(7)



______

(8)

______

II. Enclose a region with 9 dots on your dot paper. Draw the following types of triangles with end-points (vertices) on those dots:

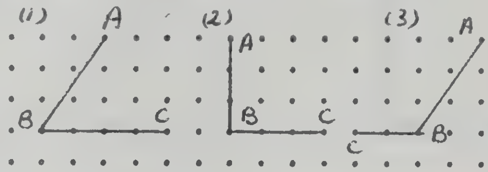
1) all possible acute △s;

2) all possible obtuse △s;

3) all possible right △s.

Use a new 9-dot region for each different triangle.

III. Copy each of the following figures onto your dot paper.



For each of these figures, draw an acute, an obtuse and a right angle at point A. Connect C to all these new points. Then try to answer the following questions:

(a) What kind of angle is ABC?

1) ______

2) ______

3) ______

(b) What are the three figures you draw?

1) ______, ______, ______.

2) ______, ______, ______.

3) ______, ______, ______.

IV. Place T(true) or F(false) beside each of these statements:

1) An acute triangle has only two acute angles. ______

2) A triangle with only one obtuse angle is called an obtuse triangle. ______

3) An acute triangle has three acute angles. ______

4) A right triangle contains only one right angle. ______

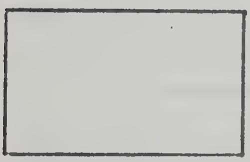
5) An obtuse triangle has one acute angle and two obtuse angles. ______

[illegible]

[illegible]

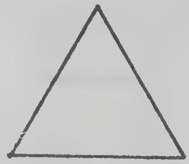
INVENTIVE EXERCISES LESSON NO. 8

I. Examine the following polygons. Write the name for each figure under the picture. Circle the ones that are regular.

(1)

____

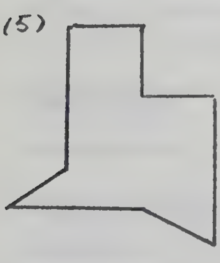
(2)

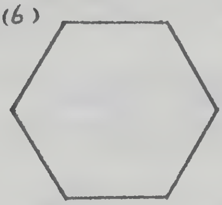
____

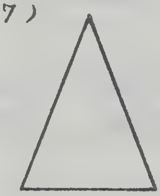
(3)

____

(4)

____

(5)

____

(6)

____

(7)

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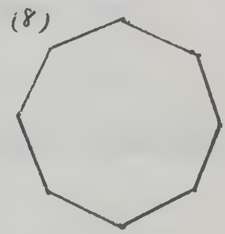
(8)

____

II. Write in your own words what is a:

1) Diagonal ______________________________

2) Regular polygon ______________________________

III. How many diagonals are possible in each of the polygons in question (I)?

1) ___, 2) ___, 3) ___, 4) ___, 5) ___, 6) ___, 7) ___, 8) ___,

9) a 10-sided polygon: ______________________________.

IV. Triangles can be formed in a polygon by joining any three vertices of the polygon. Show different ways of forming such triangles (which do not overlap with one another) in a pentagon (figure 2 in question I).

V. Different types of polygons can be formed in a 9-dot region:

3-sided:  4-sided:  5-sided: 

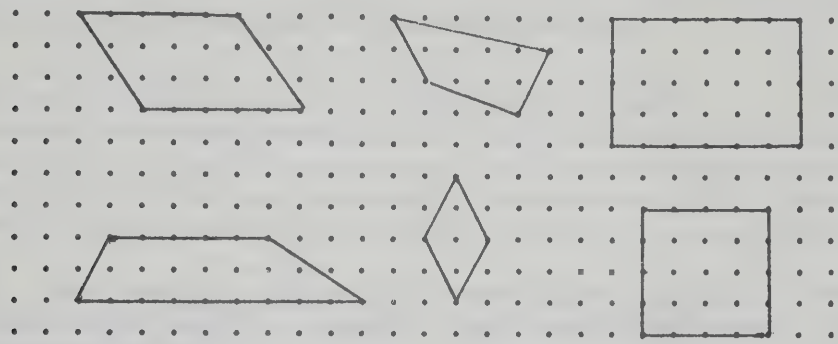
1) In a 9-dot region, we can draw polygons with ____________ sides.

2) In a 16-dot (4x4) region, we can draw polygons with __________ sides.

INVENTIVE EXERCISES

LESSON NO. 9

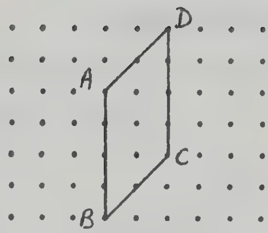
I.



Copy these onto your dot paper, draw the diagonals, fill in the chart below. Tracing paper may help you decide on the answer. Place "yes" or "no" in each column.

PROPERTIES	Quadri-laterals	Trape-zoids	Parallel-ograms	Rect-angles	Rhombi	Squares
Has two diagonals	Yes					
Diagonals bisect each other		No				
Have ½-turn symmetry about intersection of diagonals						
Diagonals form two pairs of $\cong$ triangles						
Both diagonals same length						
Diagonals form 4 $\triangle$s						
Diagonals are lines of symmetry						
Diagonals bisect each other into 4 $\cong$ segments						
Have ¼-turn symmetry about intersection of diagonals						

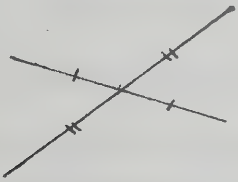
II. Copy parallelogram ABCD onto your dot paper. Draw in all the diagonals. Label the point of intersection O.



List all the properties of the parallelogram:

III. Here are the diagonals of some quadrilaterals. Name the type of quadrilateral and the property that each demonstrates. Do not draw the quadrilaterals.

(a)



(b)

(c)

(d)

a) Name

Property

b) Name

Property

c) Name

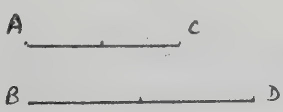
Property

d) Name

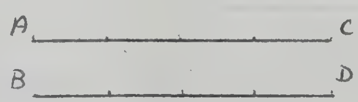
Property

IV. AC and BD are diagonals of some quadrilaterals. Put them together and construct different types of quadrilaterals.

a)



b)



AC=BD

V. On your dot paper, draw a line segment, a turn center, and the turn image, such that the original segment and its image are:

a) Opposite sides of a square.

b) Opposite sides of a parallelogram.

c) Opposite sides of a rhombus.

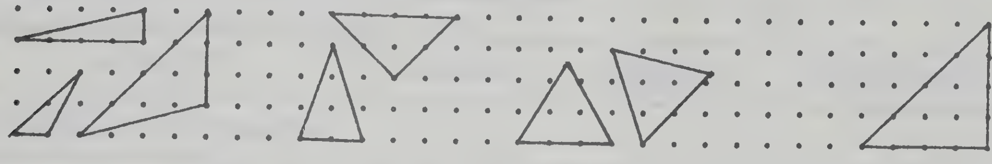
d) Opposite sides of a rectangle.

Complete the whole quadrilateral to check if you have the right quadrilateral.

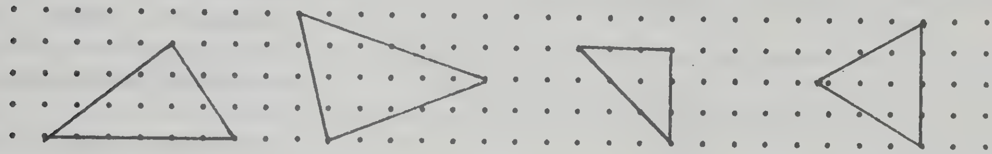
INVENTIVE EXERCISES

LESSON NO. 10

I. Classify the following triangles according to their sides:



1.______________ 2.______________ 3.______________ 4.______________



5.______________ 6.______________ 7.______________ 8.______________

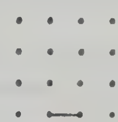
II. Write the definitions for:

a) Scalene △: ______________________________

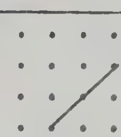
b) Isosceles △: ______________________________

c) Equilateral △: ______________________________

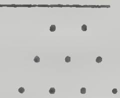
1. Draw all possible scalene △s using the given segment and placing all vertices on some of those 16 dots:



2. Draw all possible isosceles △s using the given segment and placing all vertices on some of those 16 dots:



3. Using any three of those 9 dots, draw all possible equilateral △s:
(Note the arrangement of the 9 dots.)



III. True (T) or False (F):

1. An isosceles △ is also an equilateral △. ______

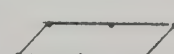
2. An equilateral △ is also an isosceles △. ______

3. A scalene △ has three sides the same length. ______

4. An isosceles △ is a △ with 2 sides the same length and the third side shorter than either of the congruent ones. ______

5. An equilateral △ has no congruent sides. ______

IV. Here is a parallelogram. Pick any 3 points on its sides and form as many △s as you can. Then answer:



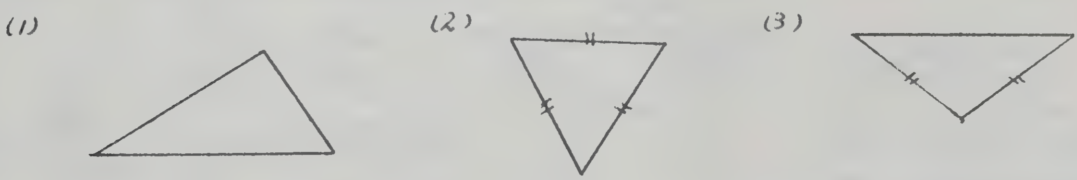
△s formed: ______ scalene △s: ______ isos. △ s: ______ equil. △s:

INVENTIVE EXERCISES LESSON NO. 11

I. Complete the following chart by making 3 △s and using tracings or paper folding to check the properties.

PROPERTY	SCALENE	ISOSCELES	EQUILATERAL
Has no lines of symmetry			
Has 1 line of symmetry			
Has 3 lines of symmetry			
Has no sides congruent			
Has 2 sides congruent			
Has 3 sides congruent			
Has no angles congruent			
Has 2 angles congruent			
Has 3 angles congruent			

II. Copy the 3 △s onto your paper. Draw in lines of symmetry if there are any. Label the new points you constructed. Fill in the chart.



TYPE OF △	PAIRS OF CONGRUENT SIDES	PAIRS OF CONGRUENT ANGLES	NUMBER OF LINES OF SYMMETRY	ANY TURN SYMMETRY?
1)				
2)				
3)				

III. You are given a 9-dot region and asked to draw △s of the following types. Place all vertices on dots.

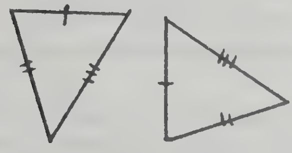
1) All possible Scalene △s.

2) All possible Isosceles △s.

Use a new 9-dot region for each different △s.

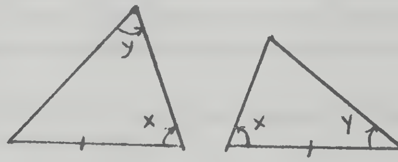
I. Using the 3 RULES of congruent △s, determine which of the following △s are congruent pairs. For those that are congruent, state the rule that guarantees this.

1)



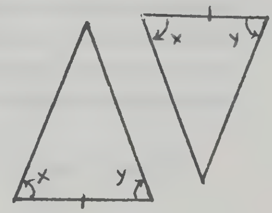
____

2)



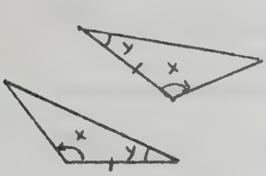
____

3)



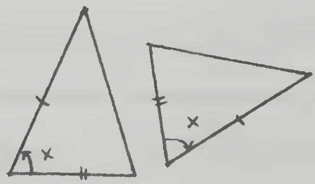
____

4)



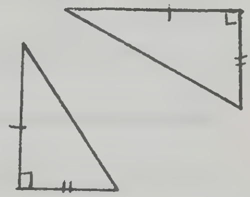
____

5)



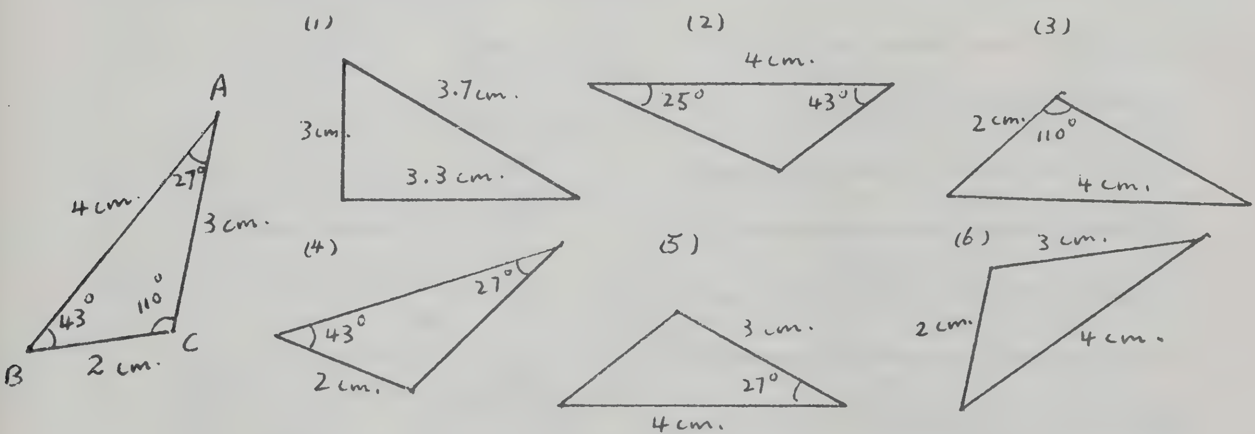
____

6)

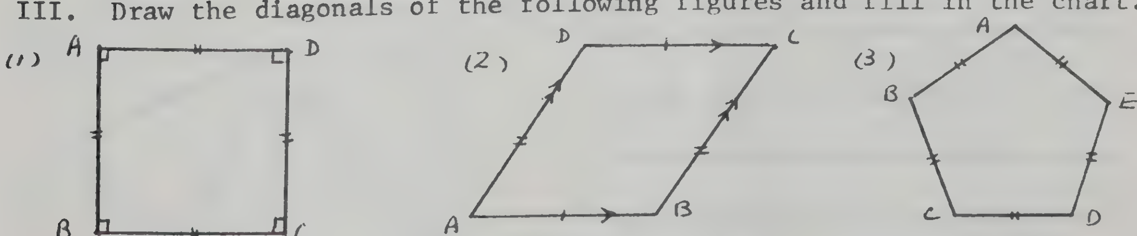


____

II. Use △ABC as a reference. Pick out all △s which are congruent to △ABC, and state the rule used.



III. Draw the diagonals of the following figures and fill in the chart.



Square

Parallelogram

Regular Pentagon

	Number of Diagonals	Name all pairs of Congruent $\triangle$s	Rule used	Total number of pairs of $\cong$ $\triangle$s.
1)				
2)				
3)				

IV. What do the following abbreviations mean? How is each useful in showing congruence of triangles?

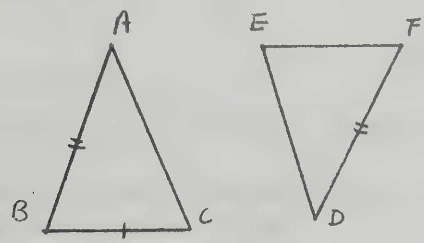
1) SSS ______________________________

2) SAS ______________________________

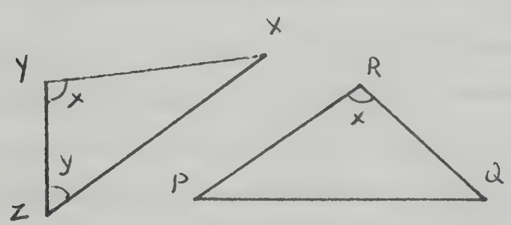
3) ASA ______________________________

V. In each question below, tell what additional information you would need to determine if the triangles are congruent or not.

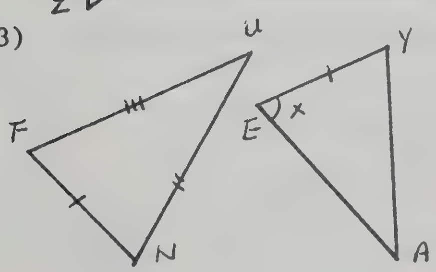
1)



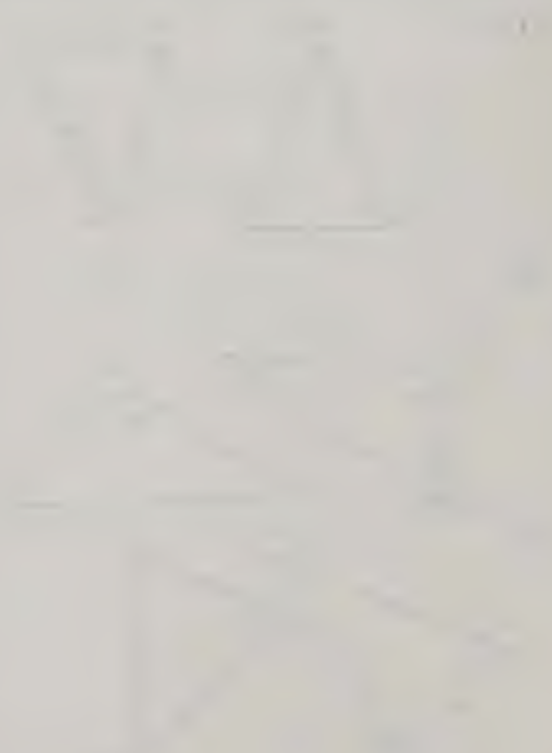
2)



3)



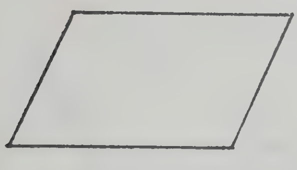
[illegible]



[illegible]

I. Classify the following polygons by writing the best name below each figure.

1)



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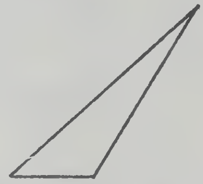
2)

____

3)



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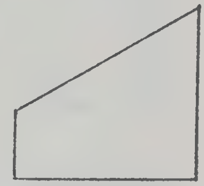
4)



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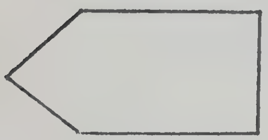
5)



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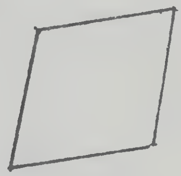

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7)

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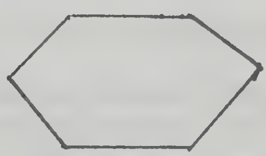

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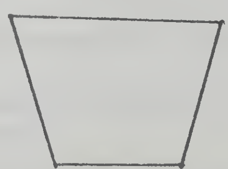
11)



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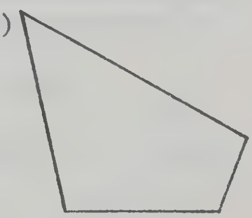
12)



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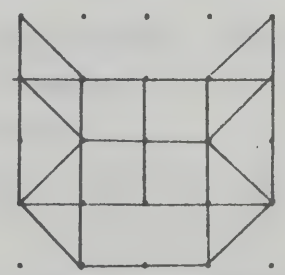
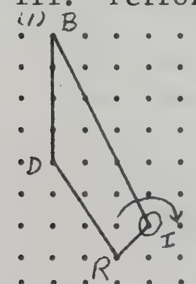
II. List all the different types of polygons you observe in this figure:

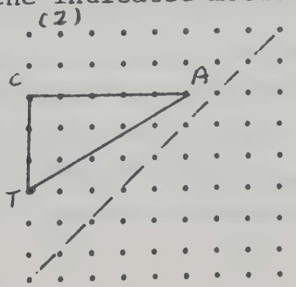


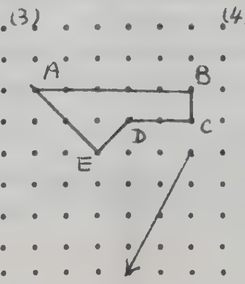
1) 8 right triangles.

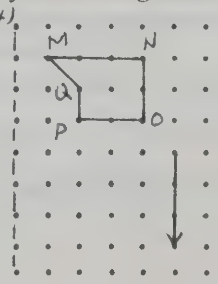
2) ________________

III. Perform the indicated motion. Be sure to label your diagrams.

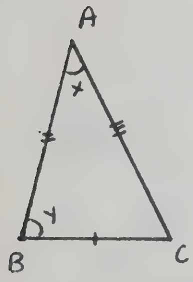




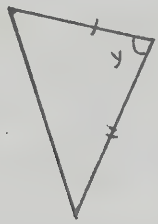




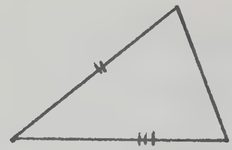
IV. Without measuring, state the △s which are congruent to △ABC together with the congruence property involved. For the rest, state the additional information needed to determine congruence.



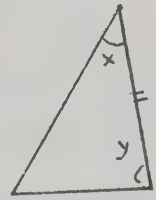
1)



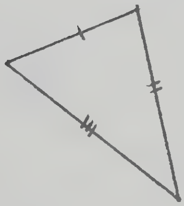
2)



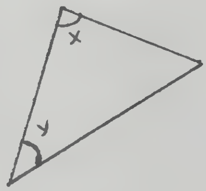
3)



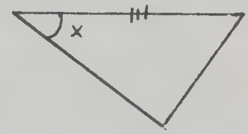
4)



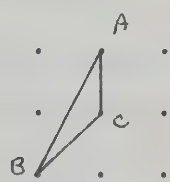
5)



6)



V. Copy the △ABC within a 9-dot region onto your dot paper.



1) Using a different region for each △, draw all the △s that are congruent to △ABC.

2) Using a new region of 9 dots for each △, draw all the NON-CONGRUENT △s, i.e. △s which are NOT congruent to △ABC, and also NOT congruent to one another.

VI. Use the best word or words to complete each statement below:

1) A _______________ △has no lines of symmetry.

2) A square has ____________ lines of symmetry.

3) Each line of symmetry for equilateral and isosceles △s divides the figure/sides/angles into 2 _______________ figure/sides/angles.

4) The angle formed by ¼-turn/½-turn on a segment is called a _______________/_____________________ angle.

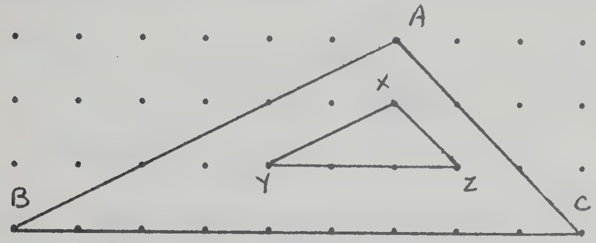
5) A polygon with 5/6/4/3 sides is a ____________/_____________/ ____________________________/__________________.

6) A quadrilateral with one/two pairs of parallel sides is called a _____________________________/_________________________.

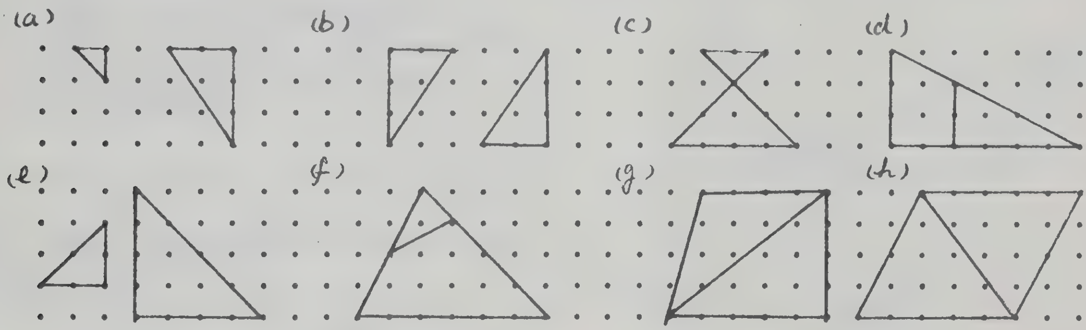
7) Lines of symmetry for ____________________ △, _________________, ______________________________ intersect at one point.

INVENTIVE EXERCISES LESSON NO. 13

I. Copy the triangles onto your dot paper and write down all the properties you notice about these two figures.



II. Use the following diagrams to answer the questions.



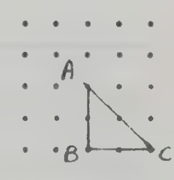
1) List those pairs of $\triangle$s which are similar: ______

2) List those pairs of $\triangle$s which are congruent: ______

3) List those pairs of $\triangle$s which are not $\cong$: ______

4) List those pairs of $\triangle$s which have 3 congruent angles but which are NOT congruent $\triangle$s: ______

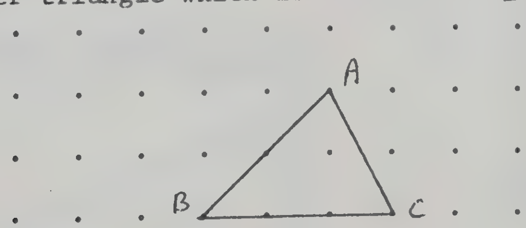
III. Form all possible reflection images of $\triangle$ABC with vertices on dots within the 25-dot region. Count these images:



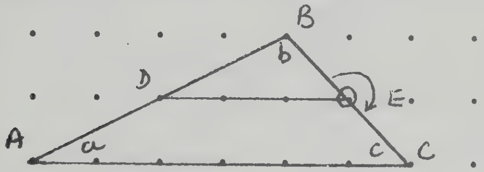
1) There are ______ $\triangle$s congruent to $\triangle$ABC.

2) There are ______ $\triangle$s similar to $\triangle$ABC.

IV. Perform a slide, or a turn, or reflection, or slide reflection, or a combination of glides on $\triangle$ABC, such that the images and $\triangle$ABC form a larger triangle which is similar to $\triangle$ABC.



I. Copy the figure onto your dot paper.

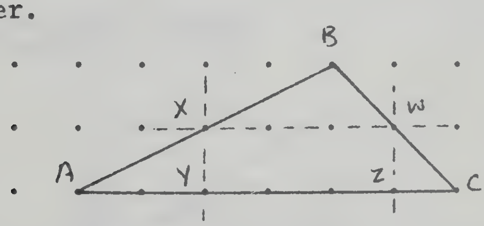


1) Slide ACED (6R, 0) and draw the image. Then draw the ½-turn image of △BDE.

2) Locate the new positions of angle a and ∠b. What can you say about the three angles a, b, and c?

____

II. Copy the figure onto your dot paper.



1) Reflect AXY in the mirror line XY. Reflect CWZ in the line WZ. Reflect BXW in the line XW. Draw the three images.

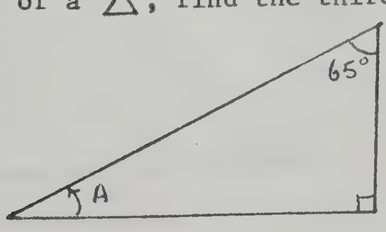
2) Draw the figure formed by these three images in a new region. What figure is this? ________

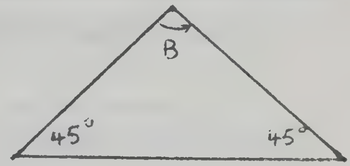
3) What happened to the angles of △ABC? ______________

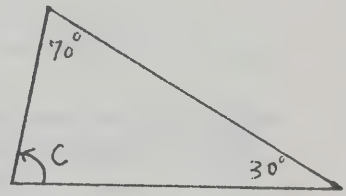
____

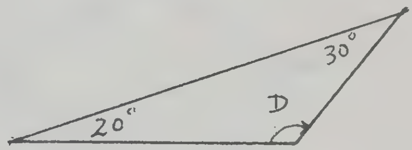
____

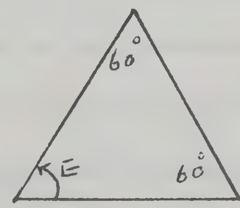
III. Given the measures of two angles of a △, find the third angle:











IV. Show how you can find the measures of angles A and B in different ways.

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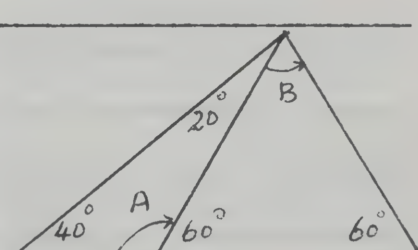

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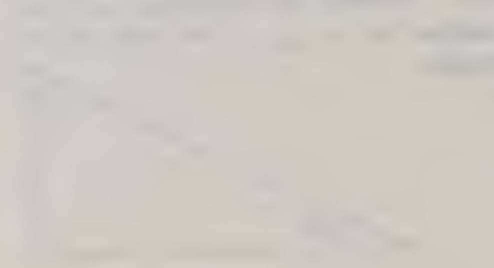
[illegible]

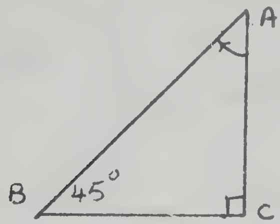
[illegible]

[illegible]

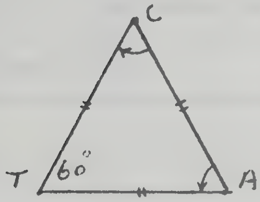


I. Determine the size, in degrees, of the missing angles.

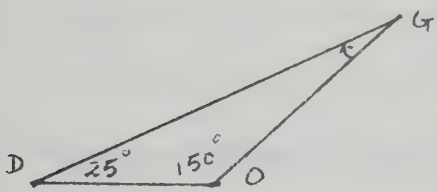
1)



2)



3)



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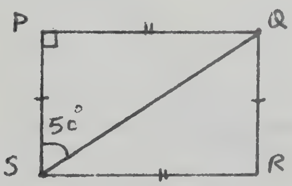
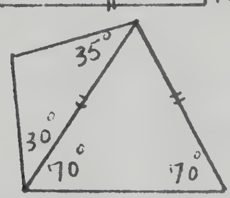
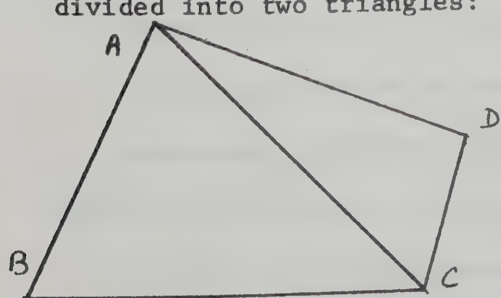


5)



Angle sum of quadrilateral PAUL is ____________.

II. Any quadrilateral can be divided into two triangles:

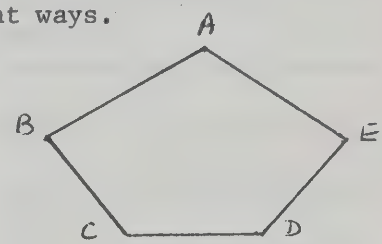


You know that the angle sum of a triangle is ____________°.

Therefore, the angle sum of the quadrilateral is:

2 x (__________°) = ______°

III. Show how you can find the angle sum of a pentagon in two different ways.



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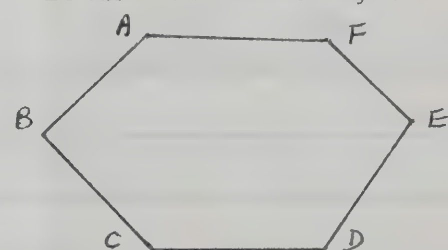
____

____

IV. 1) Angle sum of a pentagon is __________°.

2) Now you know the angle sum of a triangle, a pentagon and a quadrilateral.

Show that you can obtain the angle sum of a hexagon in MANY different ways.



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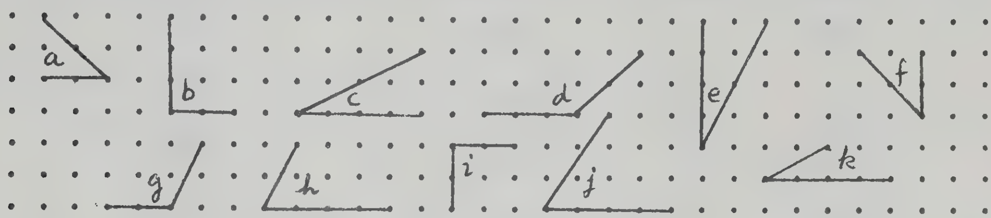
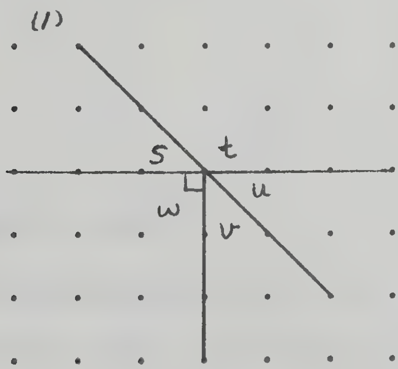
INVENTIVE EXERCISES LESSON NO. 16

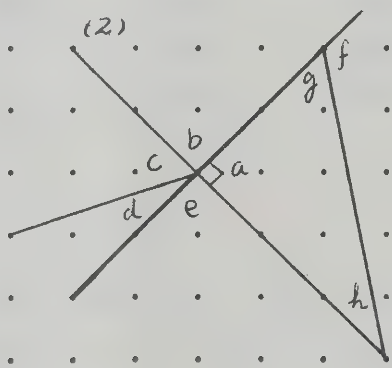
I. Fill in the following chart with the given diagrams.



Pairs of Supplementary angles	Pairs of Complementary angles
∠a and ∠d,	∠a and ∠f,

II. Using the diagrams, fill in the chart below.





	Pairs of Supplementary ∠s	Pairs of Complementary ∠s	Pairs of Opposite ∠s	Pairs of Adjacent ∠s	Pairs of ≅ ∠s	Pairs o Linear ∠
(1)	∠s, ∠t;					
(2)						

III. Place the appropriate angle in the space provided.

	Given angle	Its Supplementary Angle	Its Complementary Angle
1.	90°	$\angle$ P	No
2.	40°	_____	_____
3.	85°	_____	_____
4.	135°	_____	_____
5.	13°	_____	_____
6.	155°	_____	_____
7.	127°	_____	_____
8.	71°	_____	_____
9.	45°	_____	_____
10.	60°	_____	_____

Angles: A. 19°, B. 45°, C. 120°, D. 109°, E. 53°,
F. 27°, G. 5°, H. 50°, I. 77°, J. 167°,
K. 95°, L. 45°, M. 135°, N. 140°, O. 30°,
P. 90°, Q. 25°, R. 110°.

IV. True or False:

1) Two supplementary angles can be acute angles. _______

2) Opposite angles are linear angles. _______

3) Complementary angles must be acute angles. _______

4) Supplementary angles must be adjacent angles. _______

5) Adjacent angles are always complementary angles. _______

6) Linear angles are supplementary angles. _______

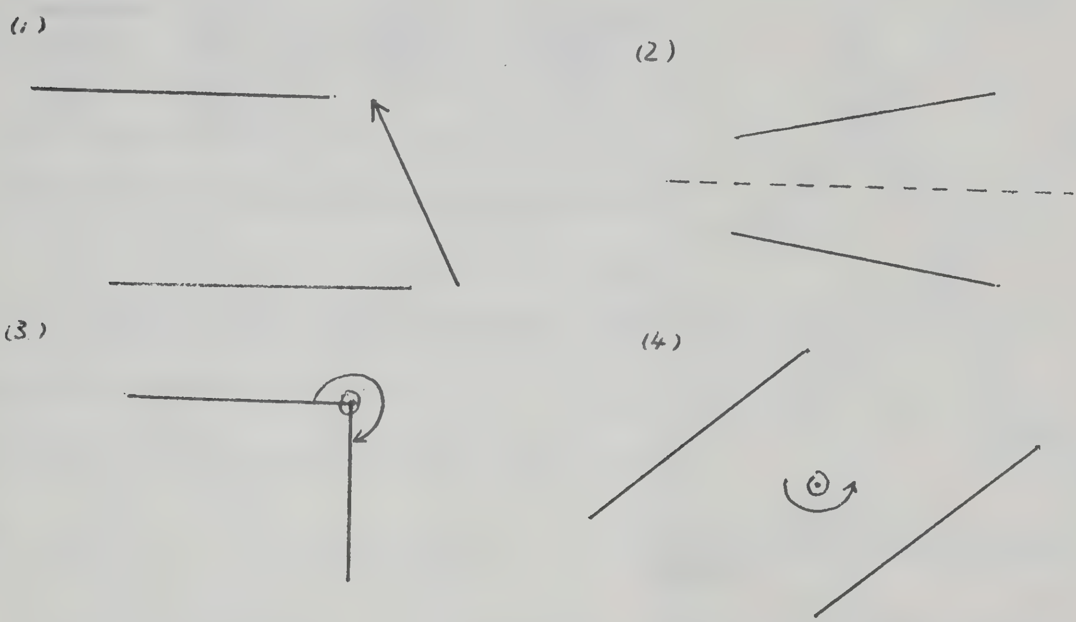
7) Linear angles are adjacent angles. _______

8) Opposite angles are formed by an angle and its ½-turn image. _______

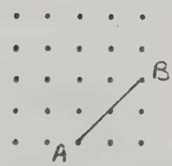
9) Two angles whose sum is a ½-turn are supplementary. _______

INVENTIVE EXERCISES LESSON NO. 17

I. Use a turn, slide, flip or slide reflection to determine which of the following are parallel.



II. You are given a 25-dot region with a segment $\overline{AB}$.



1) You are asked to draw all the possible line segments which are congruent ($\cong$) to $\overline{AB}$.

2) How many of these segments are slide images of $\overline{AB}$? __________

Indicate each slide with dotted line and put down the slide notation, (_____R/L, _____U/D).

3) How many of these segments are ½-turn images of $\overline{AB}$? __________

Indicate the turn centers together with the images.

4) How many of these segments are reflection images? __________

Draw the mirror lines for each image.

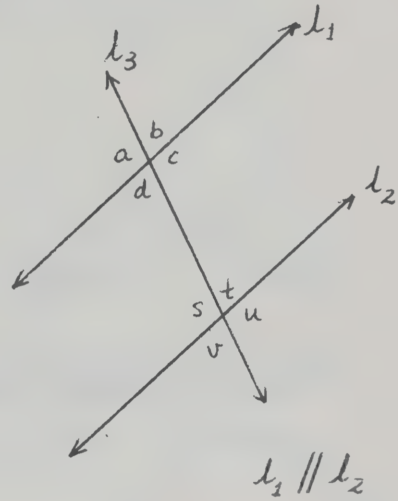
5) How many of these segments are slide-reflection images? ______

Draw the slide arrow and mirror line for each such image.

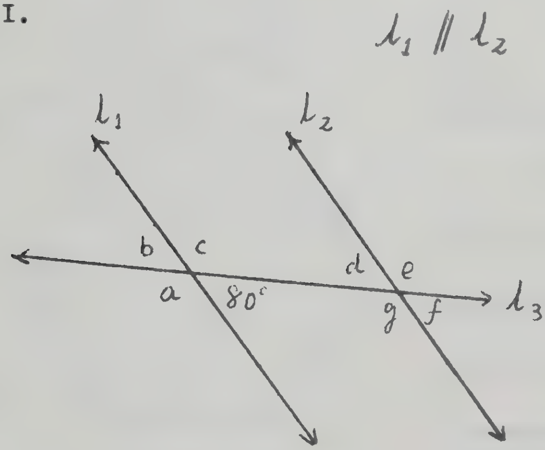
INVENTIVE EXERCISES LESSON NO. 18

I. Using the following diagram, fill in the chart:

Pairs of Corresponding Angles	Pairs of Alternative ∠s	Pairs of Opposite Angles	Pairs of Congruent Angles
∠a and __	∠d and __		∠a ≅ ∠s



II.



1) Find the measure of these angles:

∠a = ____°, ∠b = ____°,
∠c = ____°, ∠d = ____°,
∠e = ____°, ∠f = ____°.

2) Notice that:
∠a + 80° = a ½-turn.

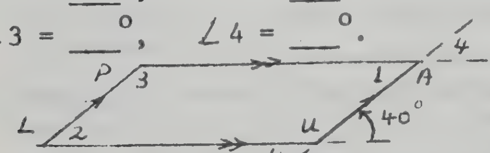
Find all other pairs of angles which produce ½-turn.

∠a + 80°; ∠a + ;

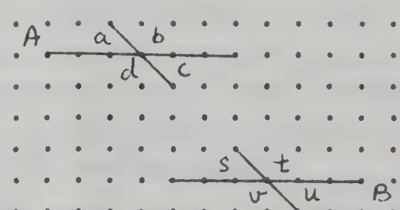
III. PAUL is a parallelogram.

1) List all pairs of parallel lines: ____________.

2) ∠1 = ___°, ∠2 = ___°,
∠3 = ___°, ∠4 = ___°.



IV. Find 2 different motions that would map cross A onto cross B. Using a new figure for each motion and draw the new positions of angles a,b,c and d.



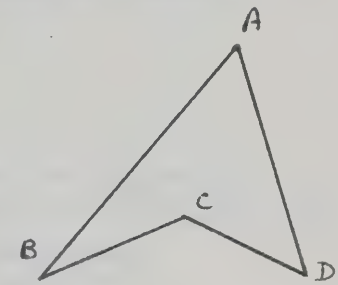
V. Construct a line // to L through point P in two different ways. Show all necessary information for each case:

INVENTIVE EXERCISES LESSON NO. 19

I. There is more than one way of finding the perimeter of this quadrilateral.

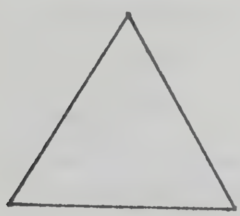
Example: Attach a string to the sides of the quadrilateral by inserting pins through the string at each vertex. Remove the string and measure its length.

Now, explain how you would go about finding the perimeter of the quadrilateral.



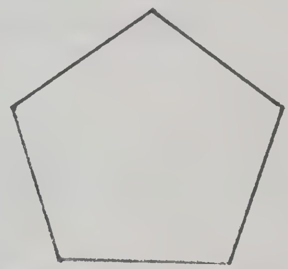
II. Here are some REGULAR polygons. Explain how you would go about finding their perimeters. Then write down their perimeters in cm.

1)

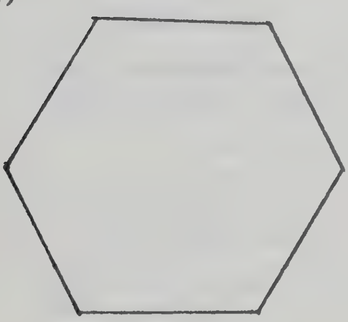


2)

3)



4)



____

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____

____

Results: 1) ____cm. 2) ____ cm. 3) ____cm. 4) ____cm.

III. You are given a string, 6 cm. in length:

1) If you want to bend the string to form triangles whose sides total the length of the string, how many different triangles can you form? Show all of them. ____________________

What kind of triangle would have the maximum area given the perimeter 6 cm.? ____________________

2) Form all possible rectangles whose sides total 6 cm. How many can you form? Show all of them. ____________________

What kind of rectangle would have the maximum area? __________

3) What other kinds of polygons can you form given the perimeter 6 cm.? ____________________

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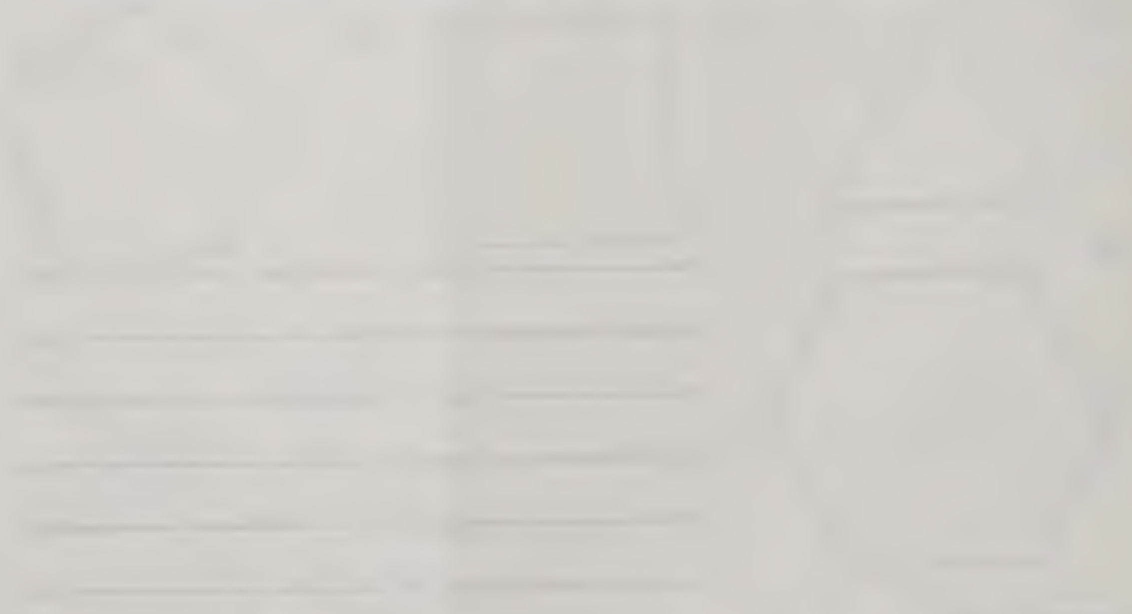
[illegible]



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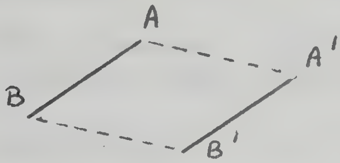
SUGGESTED SOLUTIONS FOR INVENTIVE EXERCISES

These solutions are only suggested guidelines for appropriate solutions. Teacher should feel free to encourage and accept any other solutions she/he deems appropriate.

LESSON NO. 1

I.



1) A', B', $\overline{A'B'}$

2) a. $\overline{AB} \parallel \overline{A'B'}$

   b. $\overline{AB} \cong \overline{A'B'}$

   c. $\overline{AA'} \parallel \overline{BB'}$

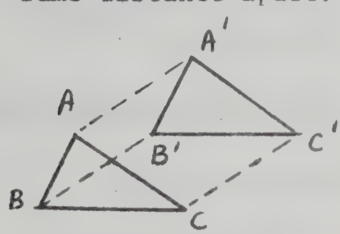
   d. $\overline{AA'} \cong \overline{BB'}$

   e. ABB'A' forms a parallelogram.

3) a. Tracing
   b. Measurement

4) Both segments are the same distance apart.

II.



a) $\overline{A'B'}$, $\overline{B'C'}$, $\overline{C'A'}$

b) 1. $\overline{AB} \parallel \overline{A'B'}$

   2. $\overline{BC} \parallel \overline{B'C'}$

   3. $\overline{CA} \parallel \overline{C'A'}$

   4. $\overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'}$

   5. $\overline{AB} \cong \overline{A'B'}$

   6. $\overline{BC} \cong \overline{B'C'}$

   7. $\overline{CA} \cong \overline{C'A'}$

   8. $\overline{AA'} \cong \overline{BB'} \cong \overline{CC'}$

   9. ABB'A', BCC'B' and ACC'A' are //grams.

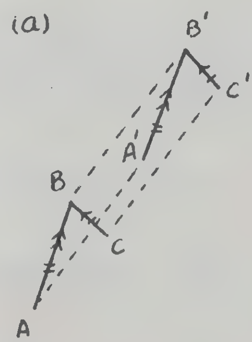
10. All angles are preserved:
    $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$

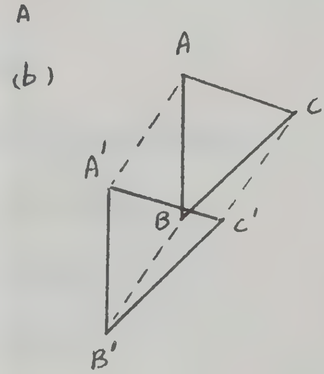
11. Shape is preserved:
    $\triangle ABC = \triangle A'B'C'$

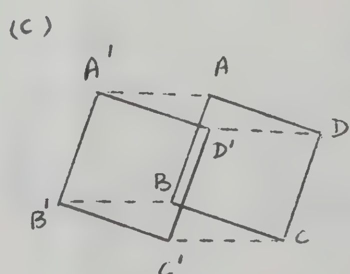
12. Orientation of $\triangle ABC$ is preserved.

13. A triangular prism is formed.

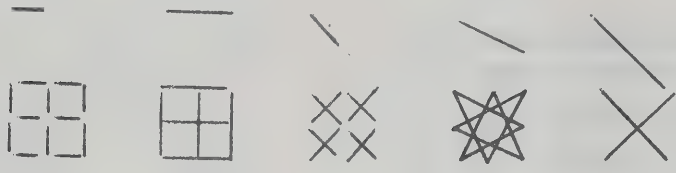
III.







IV. There are 5 different types of segments:



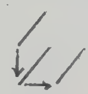
12   6   8   8   2   Total 36

(1) 36 ; (2) 8 ; (3) 4.

V. Since there is no restriction to the number of slides needed, successive slides are permissible.

1) <u>Single Slide</u> : (1 R, 1 D):

2) <u>2 Slides</u> : (0, 1 D) + (1 R, 0):



or (1 R, 0) + (0, 1 D);
(2 R, 0) + (1 L, 1 D);
(2 R, 1 D) + (1 L, 0).

3) <u>3 Slides</u>: (1 R, 0) + (1 R, 0) + (1 L, 1 D);
(1 R, 0) + (1 L, 1 D) + (1 R, 0) ; (1 R, 0) + (1 R, 1 D) + (1 L, 0);
(2 R, 0) + (0, 1 D) + (1 L, 0); (0, 1 D) + (2 R, 1 U) + (1 L, 1 D);
(0, 1 D) + (2 R, 0) + (1 L, 0).

4) <u>4 Slides</u>: (1 R, 0) + (1 R, 0) + (0, 1 D) + (1 L, 0);
and so on.

5) <u>5 Slides</u>: (1 R, 0) + (1 R, 0) + (0, 1 D) + (2 L, 0) + (1 R, 0);
and so on.

*<u>There can not be 6 slides without repetition</u>.

VI. 1) Partial Solutions:

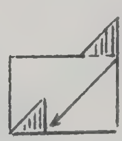
Slide the shaded part:

a) (1 R, 1 U)

or b) (2 L, 2 D)

(a)

(b)



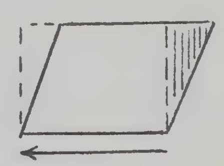
2) Partial Solutions:

Slide the shaded part:

a) (4 R, 0)
or
b) (4 L, 0)

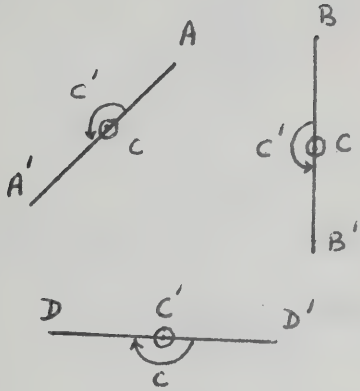
(a)

(b)

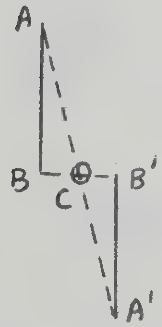


LESSON NO. 2

I.



II.



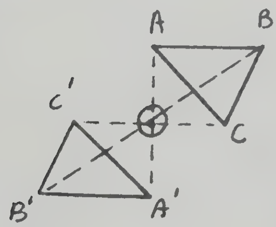
1) $\overline{AB} \parallel \overline{A'B'}$

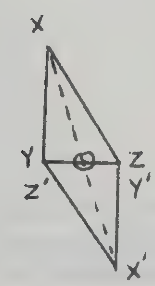
2) $\overline{AB} \cong \overline{A'B'}$

3) $\overline{AA'}$ and $\overline{BB'}$ bisect each other at the turn center.

4) Orientation of AB is reversed.

III.





1) Line segments are $\parallel$ to the image segments.

2) Line segments $\cong$ image segments.

3) $\angle$s $\cong$ image $\angle$s.

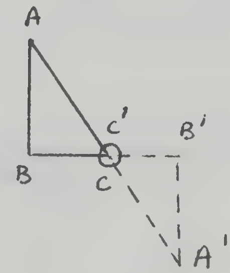
4) Shapes are preserved: original$\triangle$ $\cong$ image$\triangle$.

5) Lines joining points to images bisect one another at turn center.

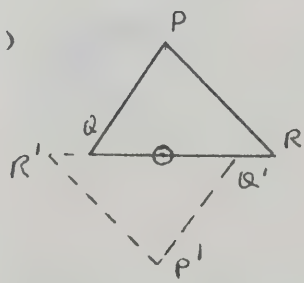
6) Orientation reversed.

IV.

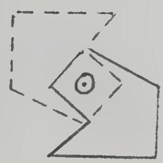
(a)



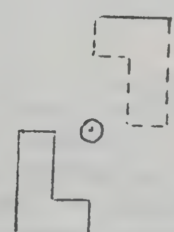
(b)



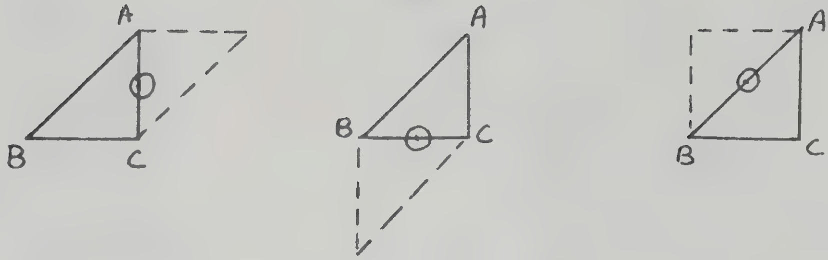
(c)



(d)



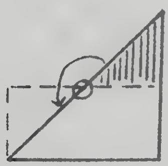
V.

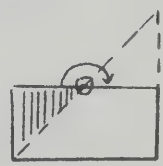


*Turn centers can not be points of the dot paper!

VI. a) 1) Perform ½-turn on shaded $\triangle$: or 2) Convert rectangle into trapezoid:

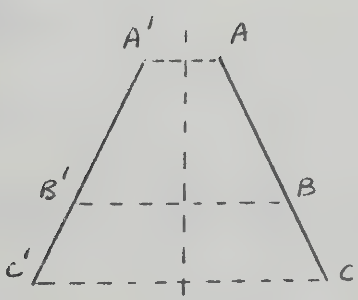
b) 1) Perform ½-turn on shaded $\triangle$: or 2) Convert rectangle into triangle:





LESSON NO. 3

I.



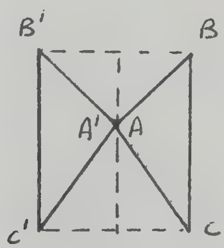
1) $\overline{AB} \cong \overline{A'B'}$

2) $\overline{AC} \cong \overline{A'C'}$

3) $\overline{BC} \cong \overline{B'C'}$

4) Mirror line is perpendicular to and bisects lines joining points to image points.

II.

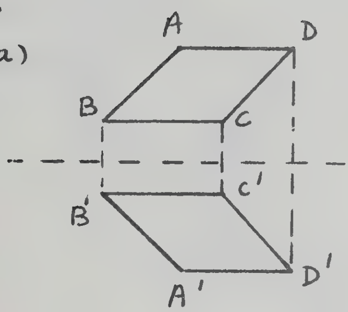


1) $\overline{AB} \cong \overline{A'B'}$; $\overline{BC} \cong \overline{B'C'}$; $\overline{CA} \cong \overline{C'A'}$.

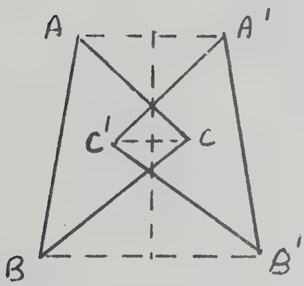
Mirror line is $\perp$ to, and bisects lines joining points to image points.
Shape is preserved:
$\triangle ABC \cong \triangle A'B'C'$.
Orientation is reversed.

II.

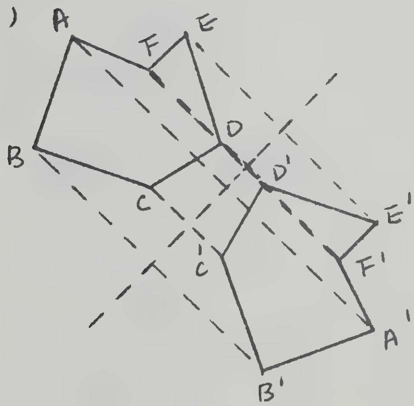
(a)



(b)



(c)

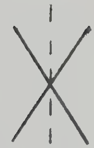


III.

(a)

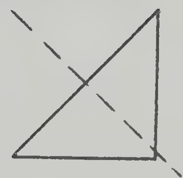
(b)

(c)

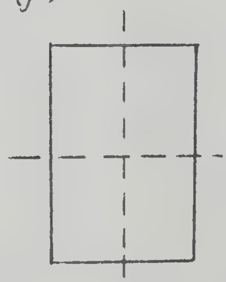


(d)

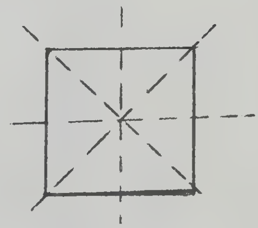
(e)



(f)



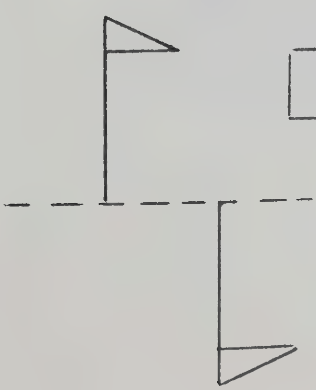
(g)



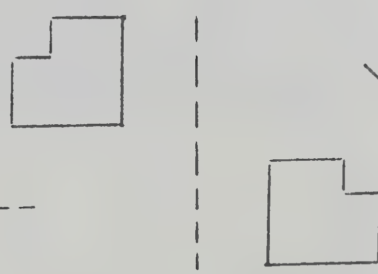
LESSON NO. 4

I.

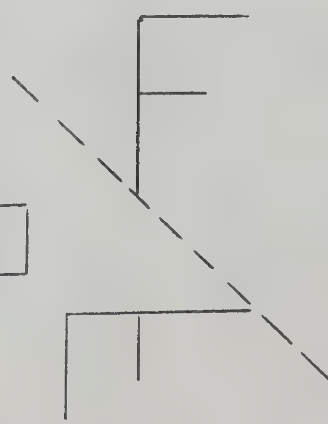
(a)



(b)

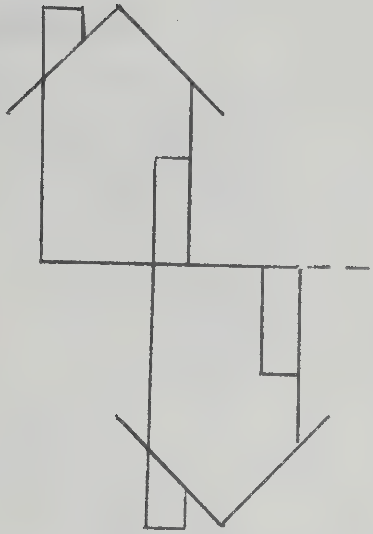


(c)



I.

(d)

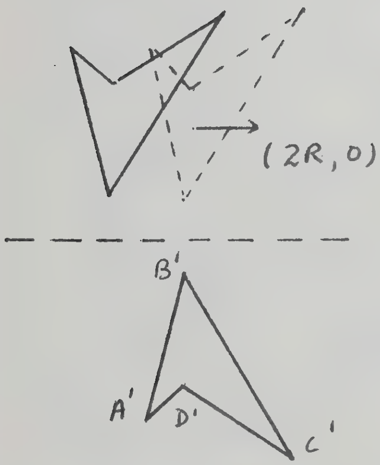


1) The original figure ≅ image figure.

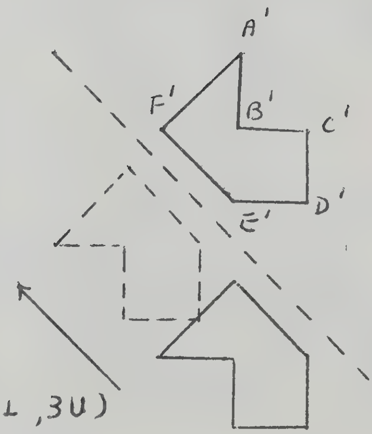
2) A slide-reflection is a combination of a slide and a reflection.
(N.B. Students can repeat all the congruence properties between original segments / angles and image segments / angles.)

II.

(a)



(b)



III. 1) Slide images and slides: 3, (7 R, 0); 7, (12 R, 3 D); 13, (8 R, 8 D); 15, (4 R, 10 D); and 16, (0, 12 D).

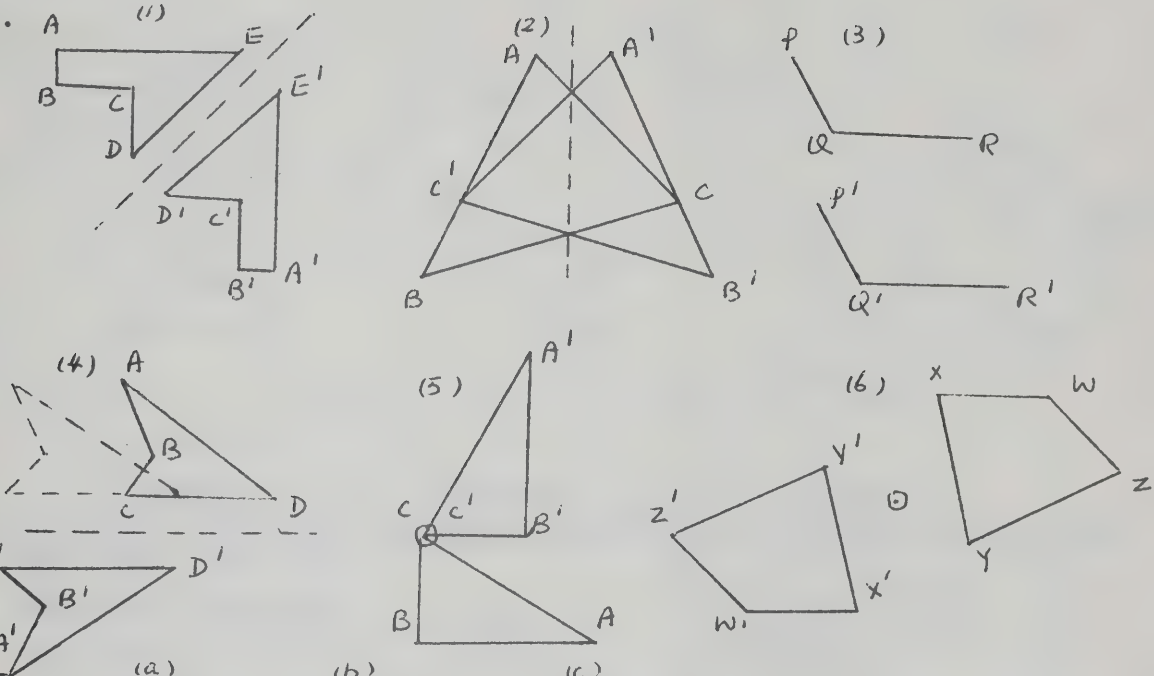
2) ½-turn images: 1; 11; 14.

3) Reflection images: 2; 4; 8; 10.

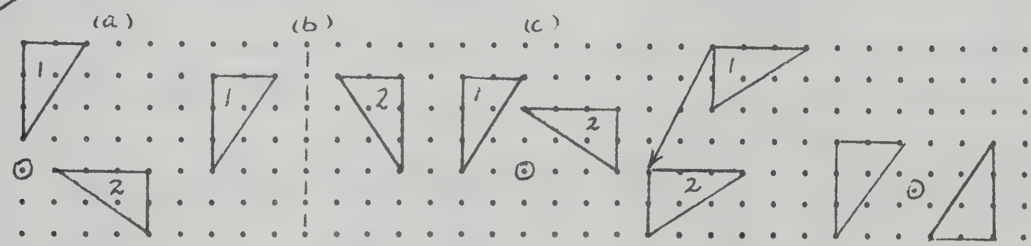
4) Slide-Reflection images: 6, (9 R, 0); 9, (0, 5 D); 18, (0, 12 D); 19, (0, 12 D); 20, (13 R, 0).

REVIEW (1-4)

I.

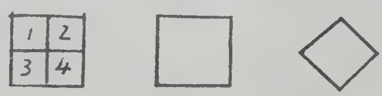


II.



¼ cw   Reflection   ¼ cw   slide   ½ cw

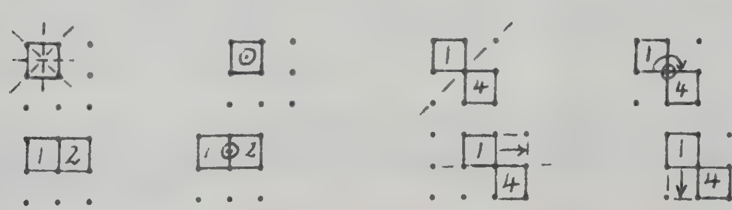
III. 1) 6 squares:



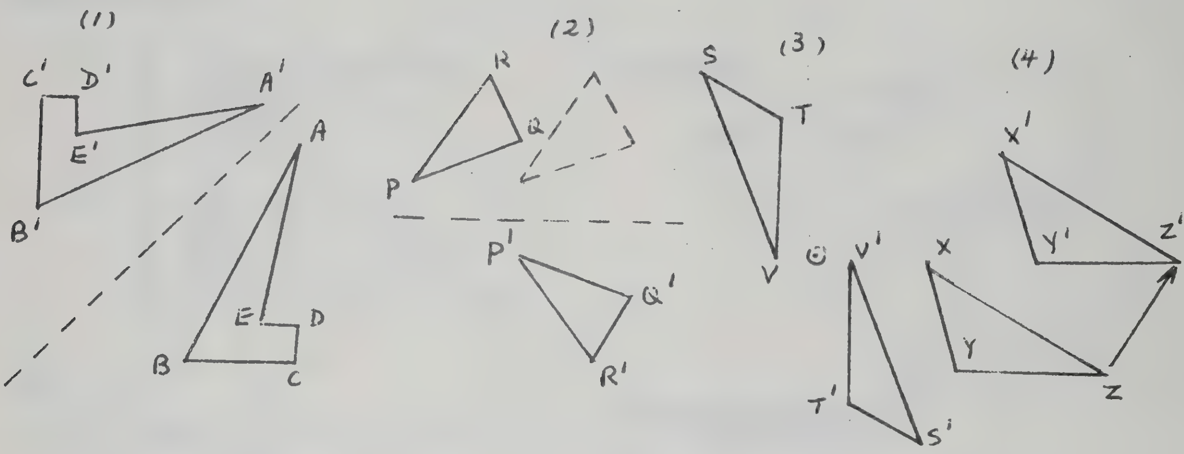
4   1   1

2) 4, including ABCD itself.

3) Slides: square 1: (0, 0); square 2: (1 R, 0);
square 3: (0, 1 D); square 4: (1 R, 1 D).
Other glides can be used, such as reflection, ½-turn and slide-reflection:

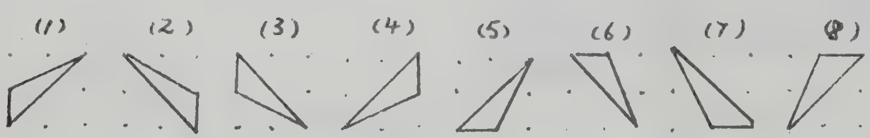


IV.



	$\cong$ segments	// segments	$\cong \angle$s
1)	$\overline{AB} \cong \overline{A'B'}$; $\overline{BC} \cong \overline{B'C'}$; $\overline{CD} \cong \overline{C'D'}$; $\overline{DE} \cong \overline{D'E'}$; $\overline{EA} \cong \overline{E'A'}$.	None	$\angle A \cong \angle A'$; $\angle B \cong \angle B'$; $\angle C \cong \angle C'$; $\angle E \cong \angle E'$; $\angle E \cong \angle D'$.
2)	$\overline{PQ} \cong \overline{P'Q'}$; $\overline{QR} \cong \overline{Q'R'}$; $\overline{RP} \cong \overline{R'P'}$.	None	$\angle P \cong \angle P'$; $\angle Q \cong \angle Q'$; $\angle R \cong \angle R'$.
3)	$\overline{ST} \cong \overline{S'T'}$; $\overline{TV} \cong \overline{T'V'}$; $\overline{VS} \cong \overline{V'S'}$.	$\overline{ST} \parallel \overline{S'T'}$; $\overline{TV} \parallel \overline{T'V'}$; $\overline{VS} \parallel \overline{V'S'}$.	$\angle S \cong \angle S'$; $\angle T \cong \angle T'$; $\angle V \cong \angle V'$.
4)	$\overline{XY} \cong \overline{X'Y'}$; $\overline{YZ} \cong \overline{Y'Z'}$; $\overline{ZX} \cong \overline{Z'X'}$.	$\overline{XY} \parallel \overline{X'Y'}$; $\overline{YZ} \parallel \overline{Y'Z'}$; $\overline{ZY} \parallel \overline{Z'X'}$.	$\angle X \cong \angle X'$; $\angle Y \cong \angle Y'$; $\angle Z \cong \angle Z'$.

V.



8 $\cong \triangle$s

1) Slide: (0, 0); 2) Reflection; 3) Reflection;

4) ½-turn about center dot; 5) Reflection; 6) ¼ CW Turn;

7) ½ CCW turn; 8) Reflection.

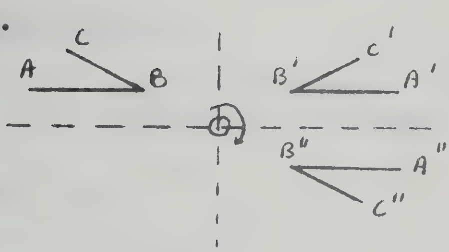
(N.B. You are given only 9 dots. You cannot glide ABC out of the region. Therefore slide [except (0, 0)] and slide reflection cannot be used.)

LESSON NO. 5

I. 1) a) Slide Reflection
b) Reflection
c) ½-turn
d) Slide

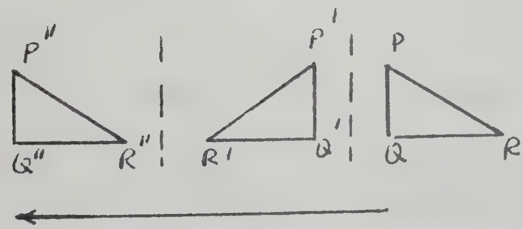
2) a) [A', B', C'];
b) [X', Y, Z', W'];
c) [P', Q', R', S'];
d) [A', B', C', D', E'].

II.



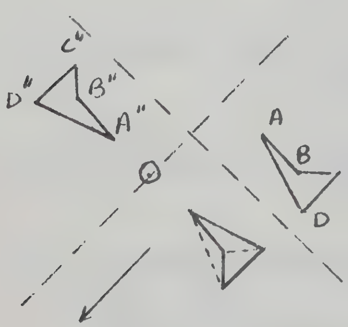
½-turn with center as intersection of mirror lines.

III.



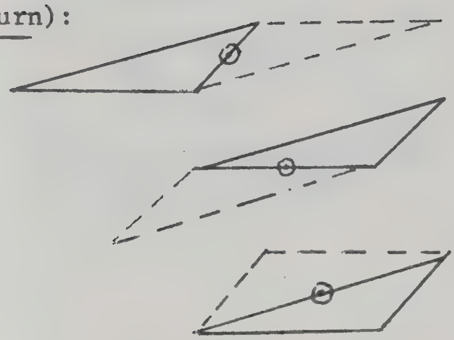
Slide: (7 L, 0).

IV.

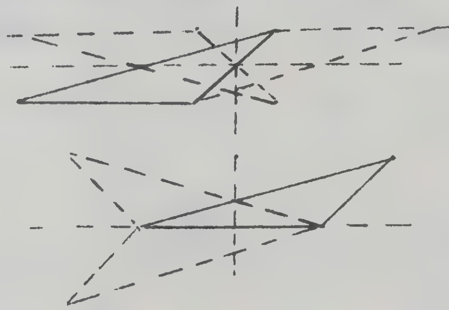


Slide Reflection.

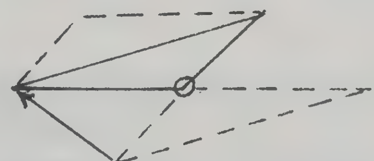
V. (½-turn):



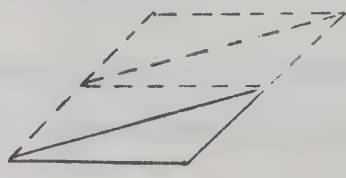
(Double-reflection):



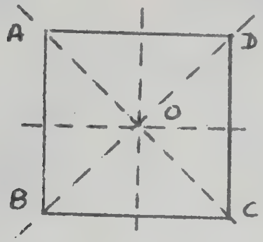
(½-turn and a slide):



In fact, infinitely many parallelograms can be formed with △PQR and its images. For example:



VI. 1) Reflection in lines of symmetry:



2) ½-turn about the center O.

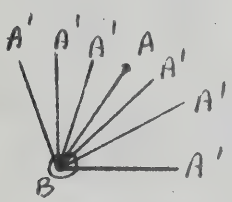
3) A revolution of 360° about any point on the plane.

4) Slide-reflection such as (5 R, 0) then reflect in D C.

LESSON NO. 6

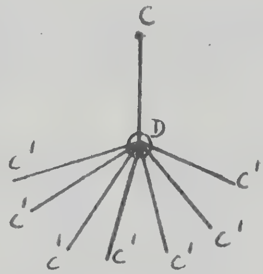
I. 1) Acute 2) Right 3) Obtuse 4) Acute 5) Right
6) Straight 7) Acute 8) Right 9) Acute 10) Straight

II. 1)



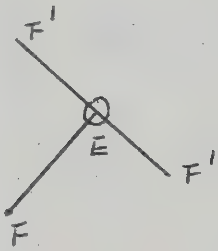
Many

2)



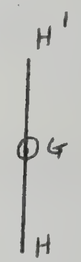
Many

3)



2

4)



1

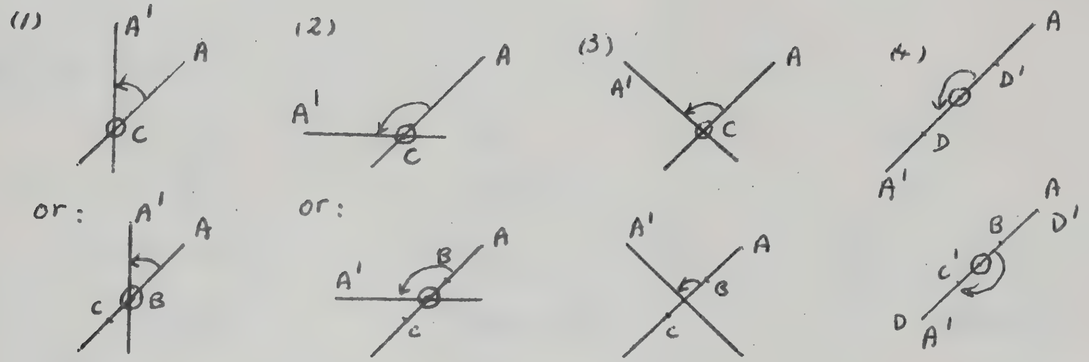
III. 1) right ∠ : ∠ formed when a line segment makes a ¼-turn about one of its end points.

2) acute ∠ : ∠ formed when a line segment makes a turn which is less than ¼-turn about one of its end points.

3) obtuse ∠ : ∠ formed when a line segment makes a turn which is less than ½ but more than ¼ about one of its end points.

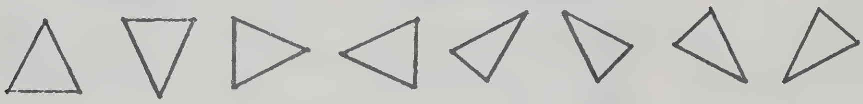
4) straight ∠ : ∠ formed when a line segment makes a ½-turn about one of its end points.

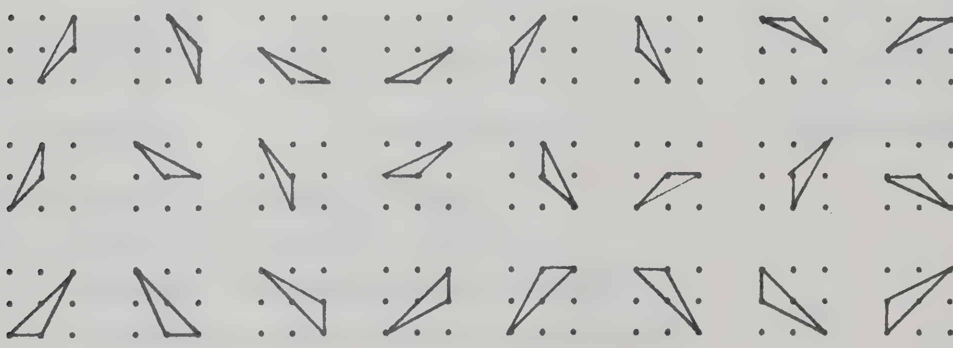
IV. N.B. Routine solutions make turn about A or D as in problem #II. above.
Creative solutions pick B or C, or any other point on ABCD. e.g.



LESSON NO. 7

I. 1) right 2) acute 3) obtuse 4) right 5) acute
6) acute 7) obtuse 8) obtuse

II. 1)  (8 acute △ s)

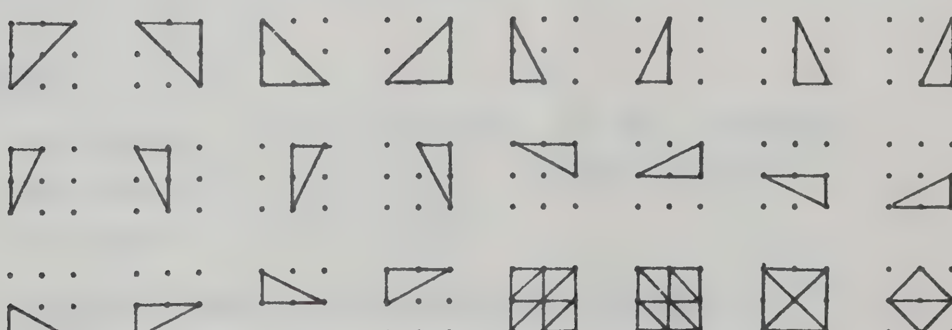
2) 

8

8

8

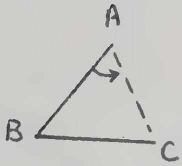
(24 obtuse △ s)

3) 

 44 right △ s: 

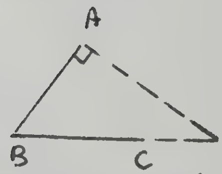
4 16 8 16

III. 1)



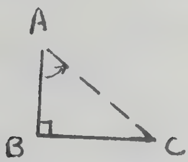
Acute △

Obtuse △

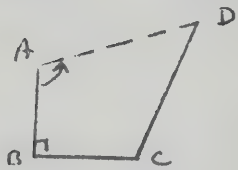


Right △

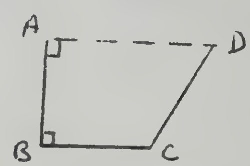
2)



Right △

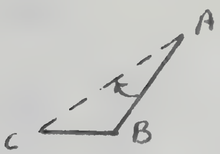


Quadrilateral

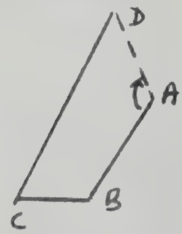


Trapezoid

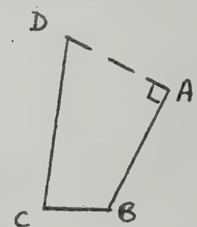
3)



Obtuse △



Quadrilateral



Quadrilateral

a) 1) acute; 2) right; 3) obtuse

b) 1) acute △ ; obtuse △ ; right △ .

2) right △ ; quadrilateral; trapezoid.

3) obtuse △ ; quadrilateral; quadrilateral.

IV. 1) F; 2) T; 3) T; 4) T; 5) F.

LESSON NO. 8

I. 1) No, rectangle; 2) Yes, pentagon; 3) Yes, triangle;
4) Yes, square; 5) No, octagon; 6) Yes, hexagon;
7) No, triangle; 8) Yes, octagon.

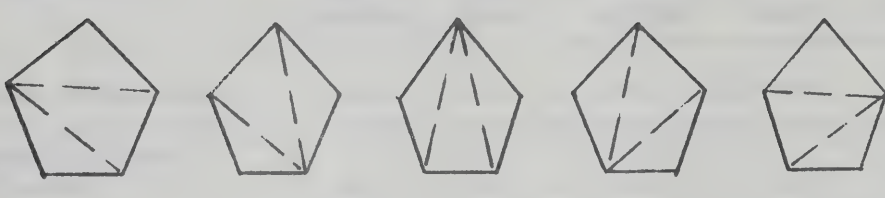
II. 1) Diagonal: Connect 2 non-consecutive vertices.
2) Regular polygon: has congruent angles and sides.

III.	Number of sides	3	4	5	6	7	8	9	10
	Number of diagonals	0	2	5	9	14	20	27	35
	Additional diagonals		2	3	4	5	6	7	8

General formula: $\frac{1}{2}n(n-3)$, where n = number of sides.

1) 2; 2) 5; 3) 0; 4) 2; 5) 20; 6) 9; 7) 0; 8) 20; 9) 35.

IV.

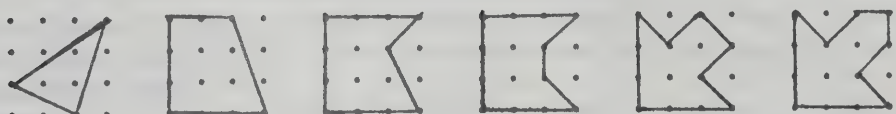


5 ways

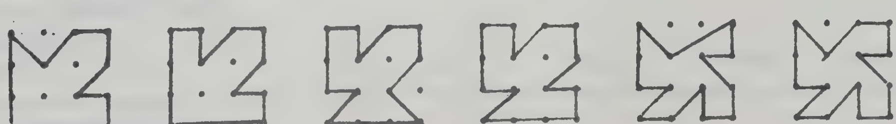
V. 1) 9 dots:

sides: 3 4 5 6 7

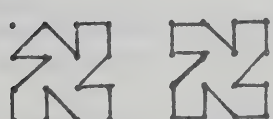
2) 16 dots:



sides: 3 4 5 6 7 8



9 10 11 12 13 14



15 16

LESSON NO. 9

I.

PROPERTIES	Quadrilaterals	Trapezoids	Parallelograms	Rectangles	Rhombi	Squares
Has two diagonals	Yes	Yes	Yes	Yes	Yes	Yes
Diagonals bisect each other	No	No	Yes	Yes	Yes	Yes
Have ½-turn symmetry about intersection of diagonals	No	No	Yes	Yes	Yes	Yes
Diagonals form two pairs of $\cong$ triangles	No	No	Yes	Yes	Yes	Yes
Both diagonals same length	No	No*	No	Yes	No	Yes
Diagonals form 4 $\triangle$s	Yes	Yes	Yes	Yes	Yes	Yes
Diagonals are lines of symmetry	No	No	No	No	Yes	Yes
Diagonals bisect each other into 4 $\cong$ segments	No	No	No	Yes	No	Yes
Have ¼-turn symmetry about intersection of diagonals	No	No	No	No	No	Yes

*"No" in general.

II. 1) $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$. (opp. sides $\cong$ and $\parallel$.)

2) $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$.

3) $\triangle ABC \cong \triangle CDA$; $\triangle ABD \cong \triangle CDB$.

$\triangle ABO \cong \triangle CDO$; $\triangle ADO \cong \triangle CBO$.

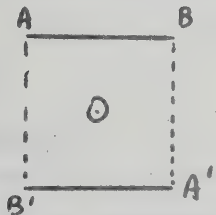
4) A parallelogram and one of its diagonals always produce 2 $\cong$ $\triangle$s.

5) ½-turn symmetry about 0.

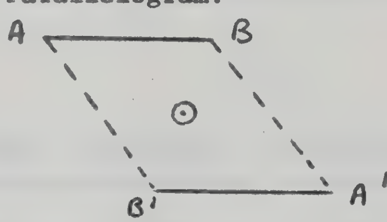
III. a) Parallelogram: diagonals bisect each other.
b) Rhombus: diagonals bisect each other at right $\angle$s.
c) Rectangle: diagonals bisect each other into 4 $\cong$ segments.
d) Square: ¼-turn symmetry.

IV. a) Quadrilateral; parallelogram; rhombus; trapezoid.
b) Quadrilateral; trapezoid; rectangle; rhombus; square.
N.B. Using the properties in problem #I. about diagonals.

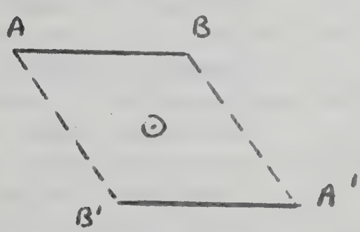
V. a) Square:



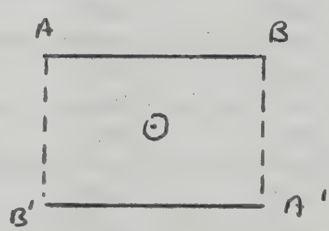
b) Parallelogram:



c) Rhombus:



d) Rectangle:



LESSON NO. 10

I. 1) Scalene 2) Isosceles 3) Equilateral 4) Isosceles
5) Scalene 6) Scalene 7) Isosceles 8) Equilateral

II. a) Scalene: no sides congruent
b) Isosceles: 2 sides congruent
c) Equilateral: 3 sides congruent

1)

10 scalene $\triangle$s

2)

3 isosceles $\triangle$s

3)

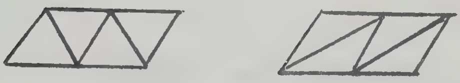
10 equilateral $\triangle$s
(8 small, 2 large)

III. (1) F; (2) T; (3) F; (4) F (can be longer); (5) F.

IV. a) 2 2 1 1

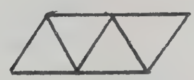
6 scalene $\triangle$ s

b) 4 4



8 isosceles $\triangle$ s

c) 4



4 equilateral $\triangle$ s

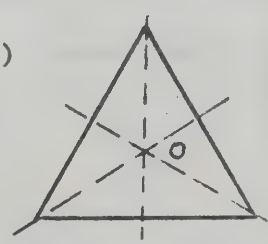
LESSON NO. 11

I.

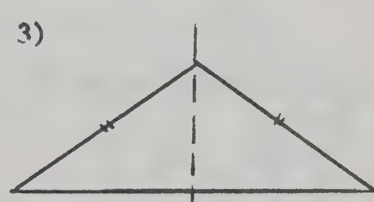
PROPERTY	SCALENE	ISOSCELES	EQUILATERAL
Has no lines of symmetry	Yes	No	No
Has 1 line of symmetry	No	Yes	Yes
Has 3 lines of symmetry	No	No	Yes
Has no sides congruent	Yes	No	No
Has 2 sides congruent	No	Yes	Yes
Has 3 sides congruent	No	No	Yes
Has no angles congruent	Yes	No	No
Has 2 angles congruent	No	Yes	Yes
Has 3 angles congruent	No	No	Yes

II. 1)

2)

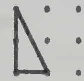


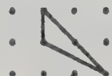
3)



	TYPE OF $\triangle$	PAIRS OF CONGRUENT SIDES	PAIRS OF CONGRUENT ANGLES	NUMBER OF LINES OF SYMMETRY	ANY TURN SYMMETRY?
1)	Scalene	None	None	None	No
2)	Equilateral	3	3	3	Yes (0)
3)	Isosceles	2	2	1	No

III. 1) The only possible Scalene $\triangle$s are: Obtuse or Right $\triangle$ s. similar solutions can be obtained as those in #II. of Objective 8:

a) Scalene right $\triangle$ s: 16 of the type: 

b) Scalene obtuse $\triangle$ s: 16 of the type: 

8 of the type: 

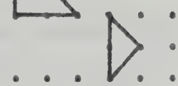
Total: 40 scalene $\triangle$ s.

2) The only possible Isosceles $\triangle$s are: Acute or Right $\triangle$ s:

a) Isosceles acute $\triangle$s: 4 of the type: 

4 of the type: 

b) Isosceles right $\triangle$s: 4 of the type: 

8 of the type: 

16 of the type: 

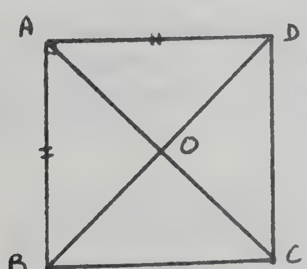
Total: 36 isosceles $\triangle$ s.

LESSON NO. 12

I. 1) Yes, SSS; 2) No; 3) Yes, ASA; 4) Yes, ASA;
5) Yes, SAS; 6) Yes, SAS.

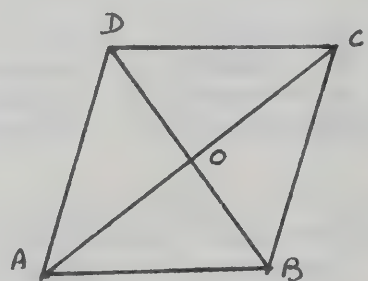
II. 1) No; 2) No; 3) No; 4) Yes, ASA;
5) Yes, SAS; 6) Yes, SSS.

III. 1)



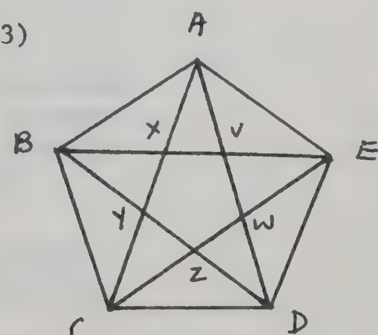
Square

2)



Parallelogram

3)



Regular Pentagon

III.

	Number of Diagonals	Name all pairs of Congruent $\triangle$s	Rule used	Total number of pairs of $\cong$ $\triangle$s	
1)	2	ABC, BCD, CDA, DAB	SSS	6	
		ABO, BCO, CDO, DAO	SSS	6	(Total = 12)
2)	2	ABC, ADC	SSS	1	
		ABD, CDB	SSS	1	
		AOB, COD	SSS	1	
		AOD, COB	SSS	1	(Total = 4)
3)	5	ABC, BCD, ...	SSS	10	
		ABD, BCE, ...	SSS	10	
		ABX, BCY, ...	SSS	10	
		ABY, ABV, ...	SSS	45	
		AXV, BXY, ...	SSS	10	
		ACW, BDV, ...	SSS	10	(Total = 95)

IV. 1) SSS: when 3 sides of one $\triangle$ are respectively equal to 3 sides of another $\triangle$, the two $\triangle$s are $\cong$.

2) SAS: when 2 sides and the $\angle$ formed by them of one $\triangle$ are respectively equal to 2 sides and the $\angle$ formed by them of a second $\triangle$, then the $\triangle$s are $\cong$.

3) ASA: when 2 $\angle$ s and the side between them of one $\triangle$ are respectively equal to 2 $\angle$ s, and the side between them of another $\triangle$, then the $\triangle$ s are $\cong$.

V. 1) $\angle B \cong \angle F$; $\overline{BC} \cong \overline{EF}$.

2) $\overline{YZ} \cong \overline{RQ}$; $\angle Z \cong \angle Q$.

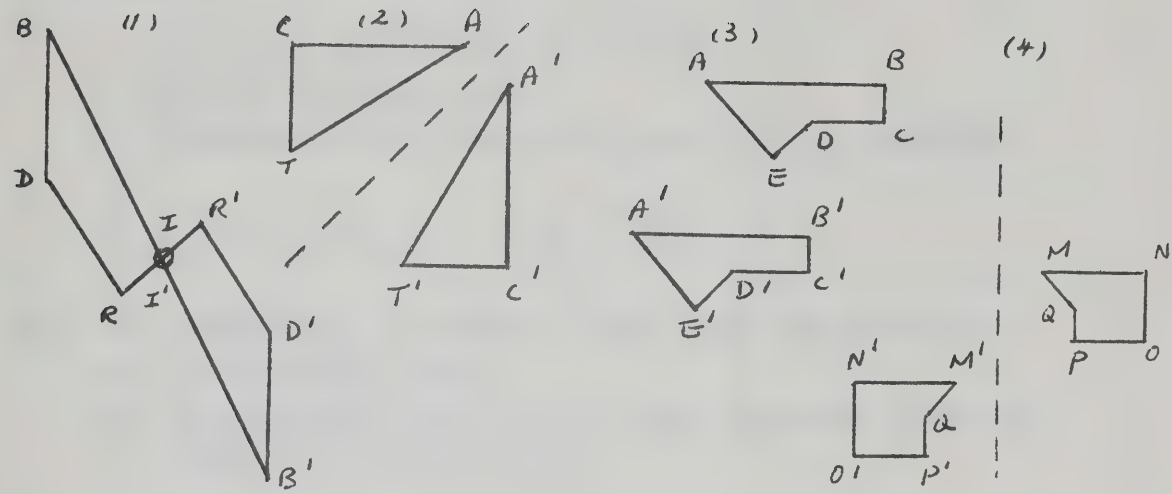
3) $\overline{FU} \cong \overline{EA}$; $\angle F \cong \angle E$, or $\angle N \cong \angle E$; $\overline{UN} \cong \overline{EA}$.

REVIEW (1-12)

I. 1) Parallelogram; 2) Square; 3) Scalene or Obtuse $\triangle$; 4) Trapezoid; 5) Pentagon; 6) Rhombus; 7) Rectangle; 8) Equilateral or Acute $\triangle$; 9) Isosceles or Acute $\triangle$; 10) Hexagon; 11) Trapezoid; 12) Quadrilateral.

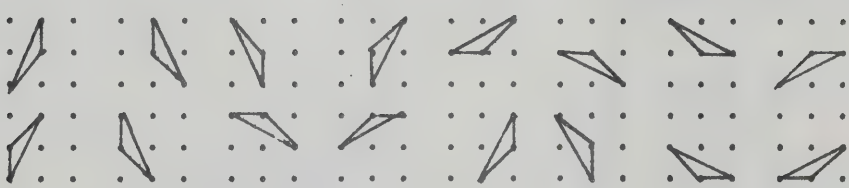
II. 1) 8 right $\triangle$s; 2) 10 isosceles $\triangle$ s; 3) 5 squares; 4) 11 rectangles; 5) 6 // grams; 6) quadrilaterals; 7) trapezoids; 8) pentagons; 9) hexagons; 10) 7 sided polygons (heptagon); and so on ...

III.



IV. 1) Yes, SAS; 2) No; 3) Yes, ASA; 4) Yes, SSS; 5) No; 6) No.

V. 1)



16 $\triangle$s $\cong$ $\triangle$ ABC

2) Non-Congruent $\triangle$s:

VI. 1) scalene; 2) four; 3) congruent; 4) right/linear (or straight); 5) pentagon/hexagon/quadrilateral/triangle; 6) trapezoid/parallelogram; 7) equilateral, square, any regular polygon.

LESSON NO. 13

I. 1) $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$;

2) $\triangle$s have the same shape;

3) Corresponding sides are proportional, but not congruent.

II. 1) b, c, d, e, h; 2) b, h;
3) a, c, d, e, f, g; 4) c, d, e.

III. N.B. Reflections of images in their sides are permissible.

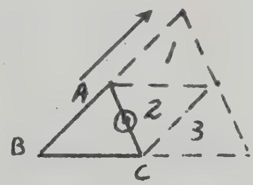
1) 36, including $\triangle$ABC.

2) 36, including $\triangle$ ABC; these images also form larger $\triangle$ s which are similar to $\triangle$ ABC.

IV. a) 1) Form $\triangle$1 by a slide (2 R, 2 U).

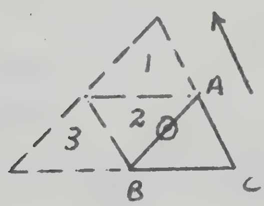
2) Form $\triangle$ 2 by ½-turn.

3) Form $\triangle$ 3 by a slide (3 R, 0).



or

b)



$\triangle$ 1: Slide (1 L, 2 U)

$\triangle$2: ½-turn

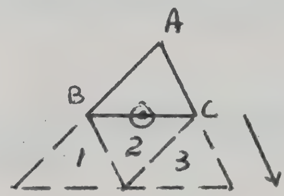
$\triangle$3: Slide (3L, 0).

or

c) $\triangle$1: Slide (2 L, 2 D).

$\triangle$ 2: ½-turn

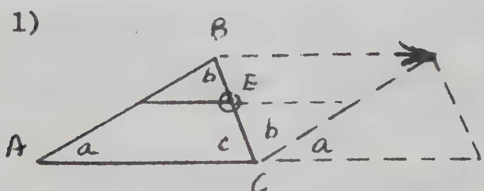
$\triangle$3: Slide (1 R, 2 D).



d) Larger similar $\triangle$ s can be formed in many ways by applying motions to the above three $\triangle$ s.

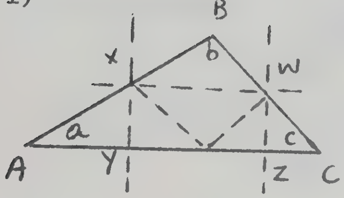
LESSON NO. 14

I. 1)

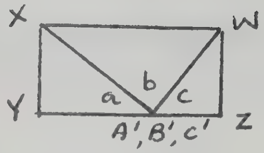


2) $\angle a + \angle b + \angle c = 180^\circ$

II. 1)



2)



A rectangle.

3) $\angle$ a + $\angle$ b + $\angle$ c = 180°
The 3 $\angle$s formed one of the straight sides of the rectangle.

III. $\angle A = 25^\circ$, $\angle B = 90^\circ$, $\angle C = 80^\circ$, $\angle D = 130^\circ$, $\angle E = 60^\circ$.

IV. $\angle A = 180^\circ - 60^\circ$
$= 180^\circ - 40^\circ - 20^\circ$

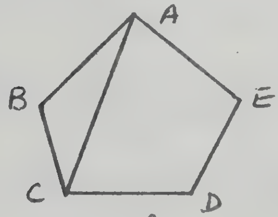
$\angle B = 180^\circ - 60^\circ - 60^\circ$
$= 180^\circ - 60^\circ - 40^\circ - 20^\circ$

LESSON NO. 15

I. 1) 45°; 2) 60°, 60°; 3) 5°; 4) 40°, 90°, 90°; 5) 100°, 115°, 40°, 360°.

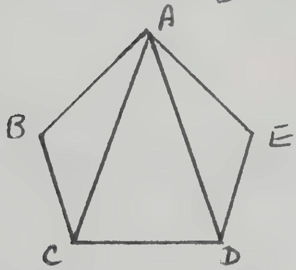
II. 180°, 180°, 360°.

III. 1)



$\angle$ sum of ABCDE
= $\angle$ sum of $\triangle$ABC + $\angle$ sum of ACDE
= 180° + 360°
= 540°.

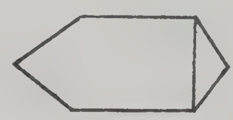
2)



$\angle$ sum of $\triangle$ABC + $\angle$ sum of $\triangle$ACD
+ $\angle$ sum of $\triangle$ADE
= 3 (180°)
= 540°.

IV. 1) $\angle$ sum of a pentagon is 540°.

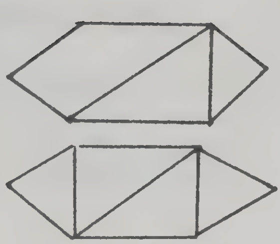
2) a) Hexagon = pentagon + triangle
= $540^\circ + 180^\circ$
= 720°



b) Hexagon = 2 (quadrilaterals)
= 2 (360°)
= 720°

c) Hexagon = 2 (triangles) + quadrilateral
= 2 (180°) + 360°
= 720°

d) Hexagon = 4 (triangles)
= 4 (180°)
= 720°



LESSON NO. 16

I.

Pairs of Supplementary Angles	Pairs of Complementary Angles
(b, i) ; (d, f) ; (g, h)	(c, h) ; (k, h)

II.

	Pairs of Supplementary $\angle$s	Pairs of Complementary $\angle$s	Pairs of Opposite $\angle$s	Pairs of Adjacent $\angle$s	Pairs of $\cong$ $\angle$s	Pairs of Linear $\angle$s
1)	(t, u) (t, v)	(u, v) (s, v)	(s, u)	(s, t);(t, u) (u, v);(v, w) (s, w)	(u, v) (s, u) (s, v)	(s, t) (t, u)
2)	(a, e) (g, f) (a, b)	(c, d) (g, h)	(b, e)	(a, b);(b, c) (c, d);(d, e) (e, a);(g, f)	(a, b) (b, e) (a, e)	(a, b) (a, e) (g, f)

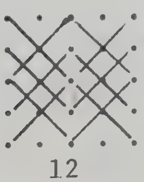
III. 2) N, H; 3) K, G; 4) B (or L), No; 5) J, I; 6) Q, No;
7) E, No; 8) D, A; 9) M, L (or B); 10) C, O.

IV. 1) F; 2) F; 3) T; 4) F; 5) F; 6) T; 7) T; 8) T; 9) T.

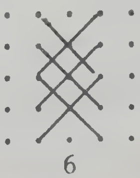
LESSON NO. 17

I. 1) Yes, Slide; 2) No, Reflection; 3) No, 3/4-turn; 4) Yes, ½-turn.

II. 1)



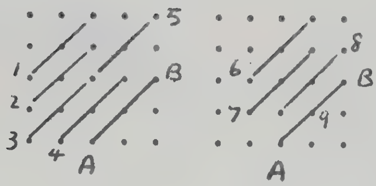
12



6

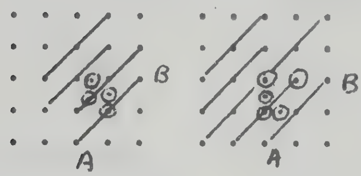
18 segments $\cong \overline{AB}$

II. 2)



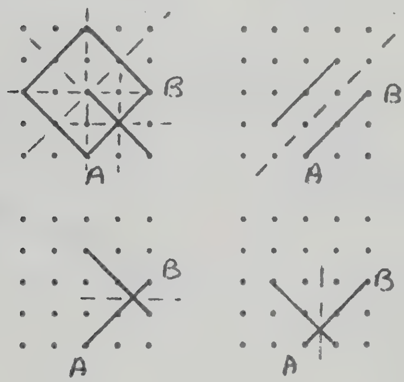
9 (including $\overline{AB}$) are slide images of $\overline{AB}$:
1) (2 R, 2 D); 2) (2 R, 1 D);
3) (2 R, 0); 4) (1 R, 0);
5) (0, 2 D); 6) (1 R, 2 D);
7) (1 R, 1 D); 8) (0, 1 D);
9) (0, 0).

3) ∵ All slide images are // to $\overline{AB}$.
They can be obtained through ½-turn too:



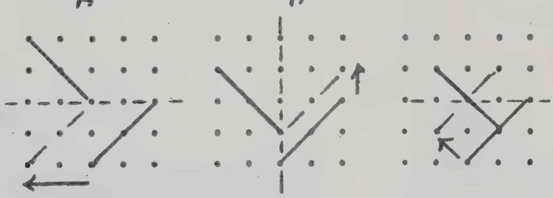
In fact, these are the only ½-turn images of $\overline{AB}$.
Total: 9 (including $\overline{AB}$)

4)



8 (including $\overline{AB}$) reflection images

5)



3 slide-reflection images

LESSON NO. 18

I.

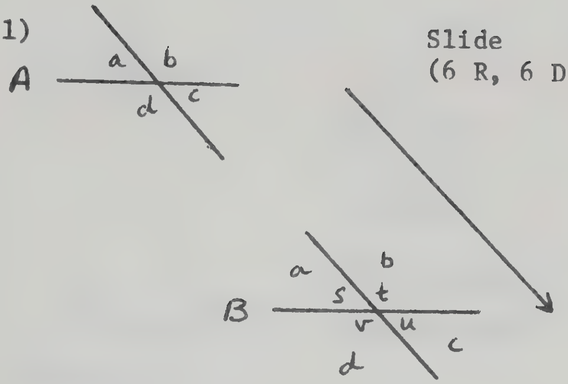
Corr. ∠s	Alt. ∠s	Opp. ∠s	≅ ∠s
(a, s);(d, v)	(d, t)	(a, c);(b, d)	(a, u);(a, c);
(b, t);(c, u)	(c, s)	(s, u);(t, v)	(b, d); (b, t); (b, v)

II. 1) 100°, 80°, 100°, 80°, 100°, 80°.

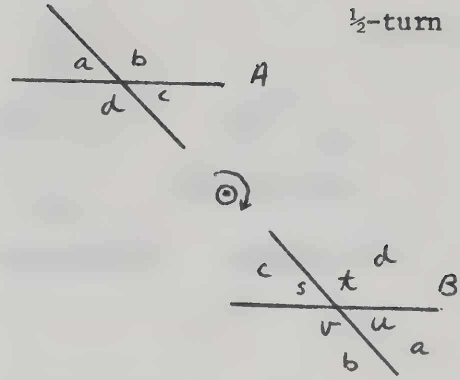
2) a + b; c + 80°; c + b; d + g; d + e; e + f; f + g;
a + f; g + 80°; b + e; c + d.

III. 1) PL // AU; PA // LU. 2) 40°, 40°, 140°, 40°.

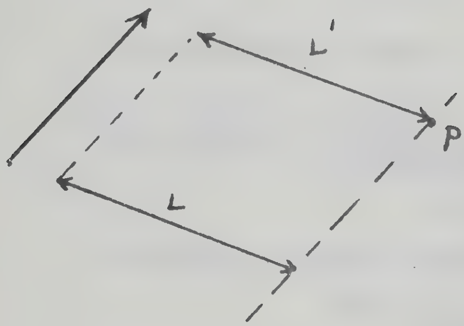
IV. 1) Slide (6 R, 6 D)



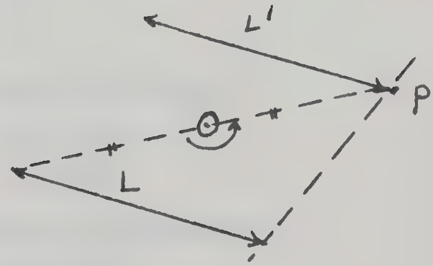
2) ½-turn



V. 1) By a Slide:



2) By a ½-turn:



<u>LESSON NO. 19</u>

I. 1) Measure each segment, and add them up:

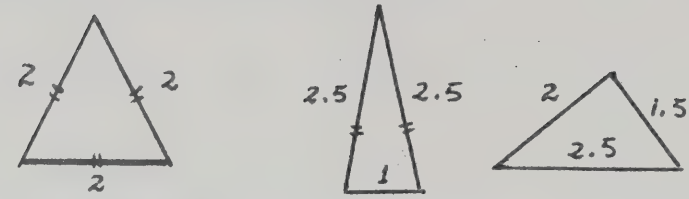
$$\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$$
$$= (4 + 2 + 1.7 + 3) \text{ cm.}$$
$$= 10.7 \text{ cm.}$$

2) Map each segment of ABCD end to end onto a straight line. Then measure the total length: $\overline{AA'} = 10.7$ cm.

II. Since all 4 figures are regular, you need only measure one side of each figure:

1) 3 x 3 = 9 cm.; 2) 4 x 3 = 12 cm.; 3) 5 x 2.2 = 11.0 cm.;
4) 6 x 2.4 = 14.4 cm.

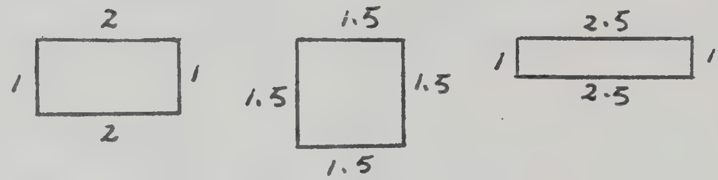
III. 1) Many examples:



Equilateral △ Isosceles △ Scalene △

Maximum area: Equilateral △

2) Many examples:



Maximum area: Square

3) All kinds of polygons. Think of regular polygons!

For example: <u>regular pentagon</u>

side = 6 ÷ 5 $\qquad\angle = 540^\circ \div 5$

= 1.2 cm. $\qquad = 108^\circ$

Similarly, n-sided regular polygon can be constructed:

$$\text{side} = \frac{6}{n}\text{ cm.}; \quad \angle = \frac{(n-2)180^\circ}{n}$$

APPENDIX B

OBSERVER RATING SCALE OF TEACHER BEHAVIOR

OBSERVER RATING SCALE OF TEACHER BEHAVIOR

Class: ______________________ Date: ______________________

		YES				NO
A.	**Teacher Omniscience**					
1.	Teacher acts as primary source of knowledge.	0	1	2	3	4
2.	Students depend on teacher to help them solve problems.	0	1	2	3	4
3.	In checking, he uses proper application of appropriate rules.	0	1	2	3	4
4.	Teacher solves problems directly without much help from the students.	0	1	2	3	4
5.	The teacher has responses corrected before proceeding.	0	1	2	3	4
B.	**Introduction of Generalization**					
1.	The teacher introduces a specific topic to the class.	0	1	2	3	4
2.	Rules are given before examples. (Ex. clarify rules)	0	1	2	3	4
3.	Teacher summarizes after each sub-topic. (e.g. problem)	0	1	2	3	4
4.	Teacher encourages students to hypothesize at solutions.	4	3	2	1	0
5.	Generalization is delayed until many students are aware of it.	4	3	2	1	0
C.	**Control of Pupil Interaction**					
1.	Students are encouraged to offer complete, correct solutions.	0	1	2	3	4
2.	Teacher questions students on direct items of the topic.	0	1	2	3	4
3.	Classroom interaction deviates from the topic.	4	3	2	1	0
4.	Students are encouraged to reveal rules that apply.	0	1	2	3	4
5.	Premature verbalization of the rule is discouraged.	4	3	2	1	0
D.	**Method of Answering Questions**					
1.	The teacher answers by reiterating and explaining the rule.	0	1	2	3	4
2.	The teacher gives examples to clarify application of rule.	0	1	2	3	4
3.	Teacher answers student's question rather than referring to class.	0	1	2	3	4
4.	The teacher answers by referring to student's computational sequence.	4	3	2	1	0
5.	The teacher uses sequenced examples without hinting the rule.	4	3	2	1	0

E.	<u>Use of Student Responses</u>	YES NO
1.	Students' suggestions are tested before comment or evaluation.	4 3 2 1 0
2.	Students evaluate each other's responses.	4 3 2 1 0
3.	Students' responses guide classroom interaction.	4 3 2 1 0
4.	Student responses bear directly on topic.	0 1 2 3 4
5.	Students give only possible hints to the solution.	4 3 2 1 0
F.	<u>Method of Eliminating False Concepts</u>	
1.	Students are warned of errors made in applying the rule.	0 1 2 3 4
2.	Students are warned of problems with special peculiarities.	0 1 2 3 4
3.	Students are cautioned about false generalizations.	0 1 2 3 4
4.	Students are given specific ways of working problems.	0 1 2 3 4
5.	Teacher leads class to overgeneralization of rule.	4 3 2 1 0

APPENDIX C

PRE-TEST

PIAGETIAN SECTIONING TEST*

(SEC)

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*All the materials of this test were constructed by Mr. Dale Drost who administered it to the two classes of this study as part of his doctoral research. With the kind permission of Mr. Drost, relevant parts of the test materials are reproduced in this Appendix.

PROTOCOL FOR ADMINISTERING THE TEST

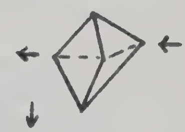
1. Distribute the answer booklets and have the students complete the first page. In the space for grade include the teacher's name as well as the grade. Some explanation may be necessary concerning the age with regard to months.

2. Show the students the complete sphere and discuss its properties briefly. Ask the students what plane shape will be seen if you cut the sphere into two parts. Illustrate with the cut sphere and draw a circle on the blackboard freehand. Have the students do the same on their paper below the information they have completed.

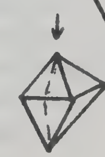
3. The students now turn to page 2 of their booklet.

4. Illustrate the complete octahedron and discuss its properties briefly. How many faces, how many edges, etc.

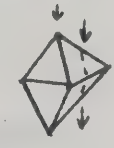
5. Perform the parallel cut on the octahedron using a ruler in place of a knife. Ask the students what plane shape would result if you were able to look at the cut pieces. Use the cut solid to indicate the resulting plane figure is a square. Have them draw freehand a square in the box labelled A on page 2. You draw one on the board.



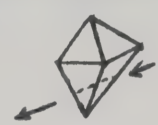
6. Repeat step 5 with a longitudinal cut. The result is a rhombus. Have the students draw a rhombus in box B.



7. Repeat step 5 with a transverse cut. The result is an hexagon. Have the students draw a hexagon in box C.



8. Repeat step 5 with an oblique cut. The result is a trapezoid. Have the students draw a trapezoid in box D.



9. Indicate to the students that in the first part of the exercise, that you will illustrate similar cuts as executed above in the following solids: cube, triangular prism, cone and parallelepiped. Show the solids briefly. Also, without the solids, indicate the general direction of the four cuts as being:
a) perpendicular to the floor and pointing at the student,
b) perpendicular to the floor and facing the student,
c) parallel to the floor,
d) oblique to the floor.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

10. Points of emphasis

a) The students are to draw the plane surface they would see if the solid were taken apart. In the exercise the cut will be illustrated but the solid not taken apart.

b) The students are to make the drawings freehand being careful to make lines approximately equal when that is the case, to make corners square when that is the case, and lines straight if they are straight lines, and parallel if they are parallel lines.

c) If the students wish the cut illustrated again, do it immediately but do not return to a cut later.

d) Make sure the cut is being drawn in the proper place on the answer sheet.

e) The tester should make the cut and turn to face the left and right of the class so every student sees the cuts face on.

11. QUESTIONS??

12. Illustrate the 16 cuts as indicated on the sheet labelled -- PRE-TEST DRAWING.

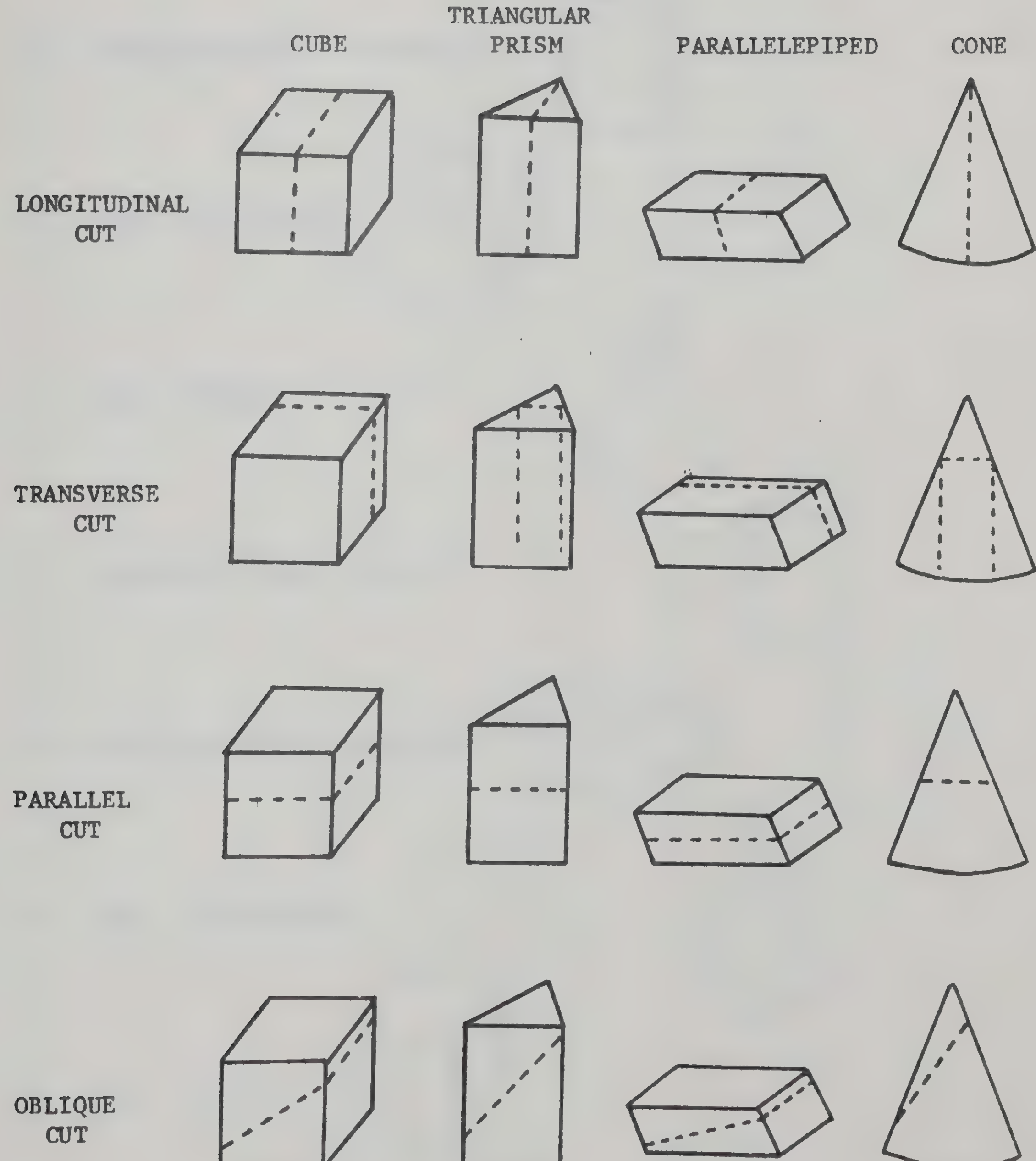
13. Distribute the multiple choice distractors. The students are not to mark on their paper. Explain that each cut will be illustrated once again and the student is to select the answer he thinks is correct and circle the appropriate letter on the answer sheet. If you think the answer is not on the sheet, circle F. Take care that the students are considering the correct set of distractors after each cut.

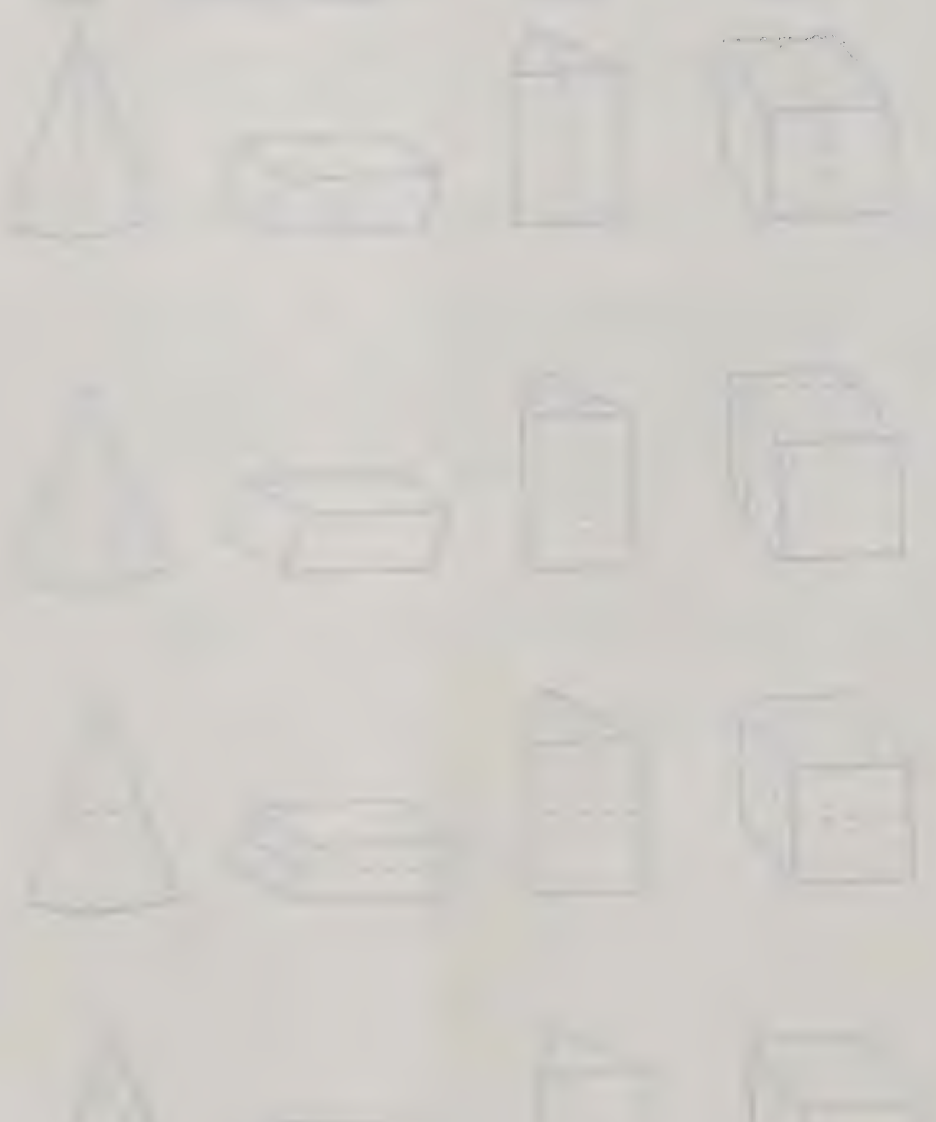
14. QUESTIONS??

15. Illustrate the 16 cuts as indicated on the sheet labelled -- PRE-TEST MULTIPLE CHOICE.

16. Collect the answer sheets.

SOLIDS AND CUTS

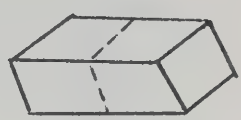




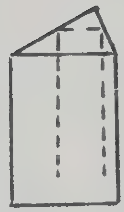
PART I: DRAWING

The cuts are presented in the following order:

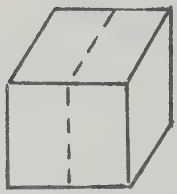
1. parallelepiped: longitudinal



2. triangular prism: transverse

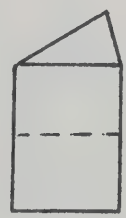


3. cube: longitudinal

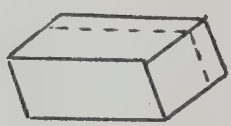


4. cone: oblique

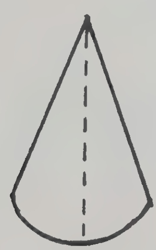
5. triangular prism: parallel



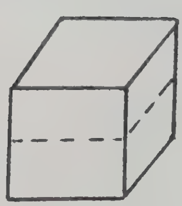
6. parallelepiped: transverse



7. cone: longitudinal



8. cube: parallel



[illegible]

[illegible]

[illegible]

[illegible]

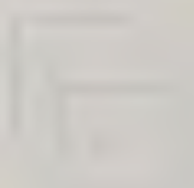
[illegible]

[illegible]

[illegible]

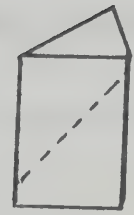
[illegible]

[illegible]

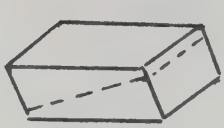


[illegible]

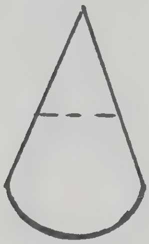
9. triangular prism: oblique



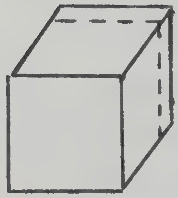
10. parallelepiped: oblique



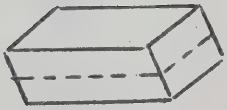
11. cone: parallel



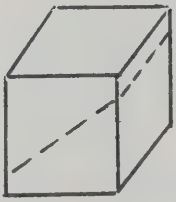
12. cube: transverse



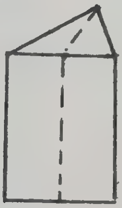
13. parallelepiped: parallel



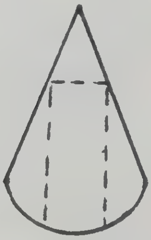
14. cube: oblique

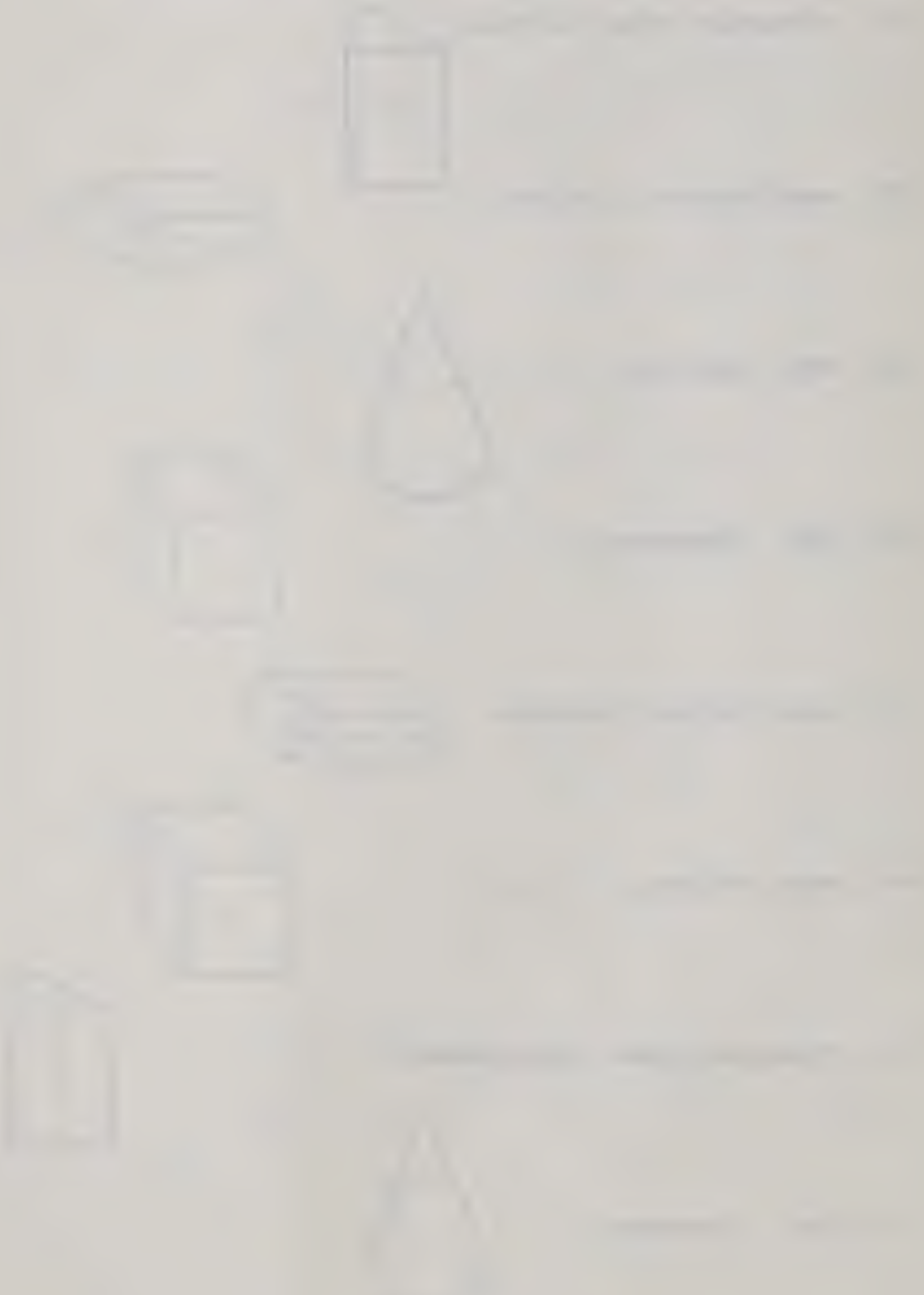


15. triangular prism: longitudinal



16. cone: transverse

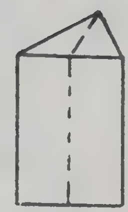




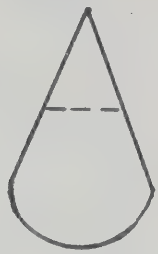
PART II: MULTIPLE CHOICE

The cuts are presented in the following order:

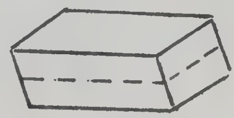
1. triangular prism: longitudinal



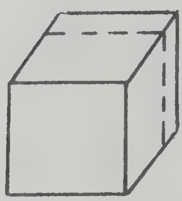
2. cone: parallel



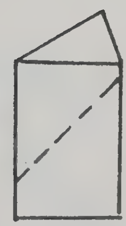
3. parallelepiped: parallel



4. cube: transverse

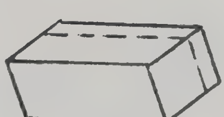


5. triangular prism: oblique

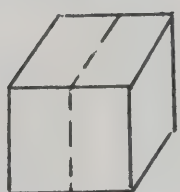


6. cone: longitudinal

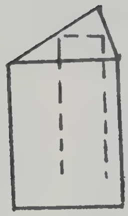
7. parallelepiped: transverse



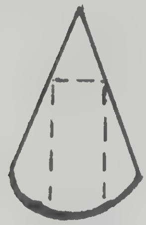
8. cube: longitudinal



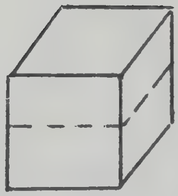
9. triangular prism: transverse



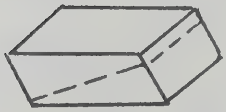
10. cone: transverse



11. cube: parallel



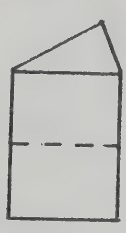
12. parallelepiped: oblique



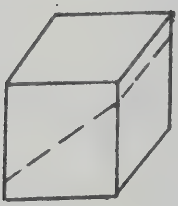
13. cone: oblique

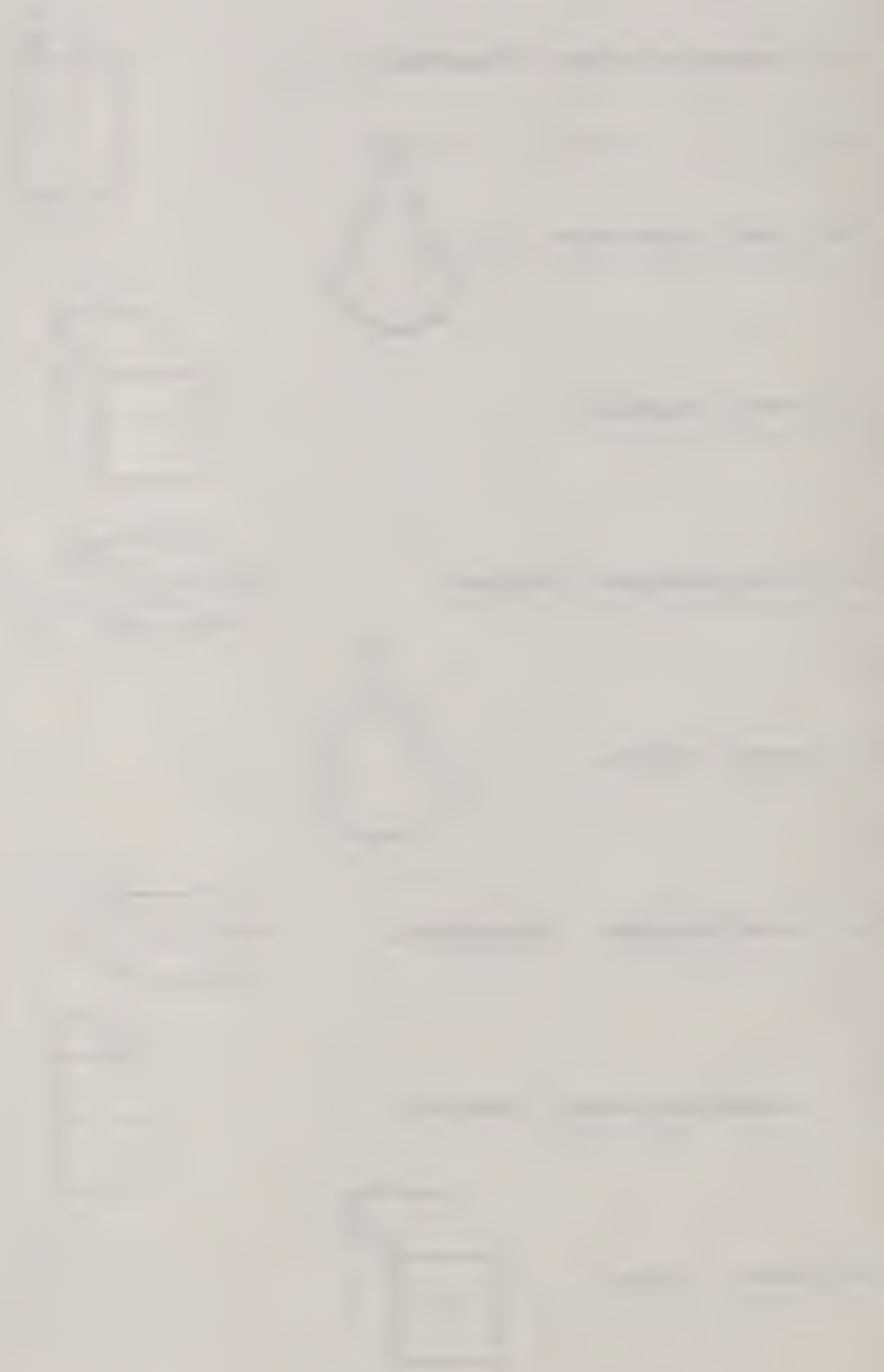
14. parallelepiped: longitudinal

15. triangular prism: parallel

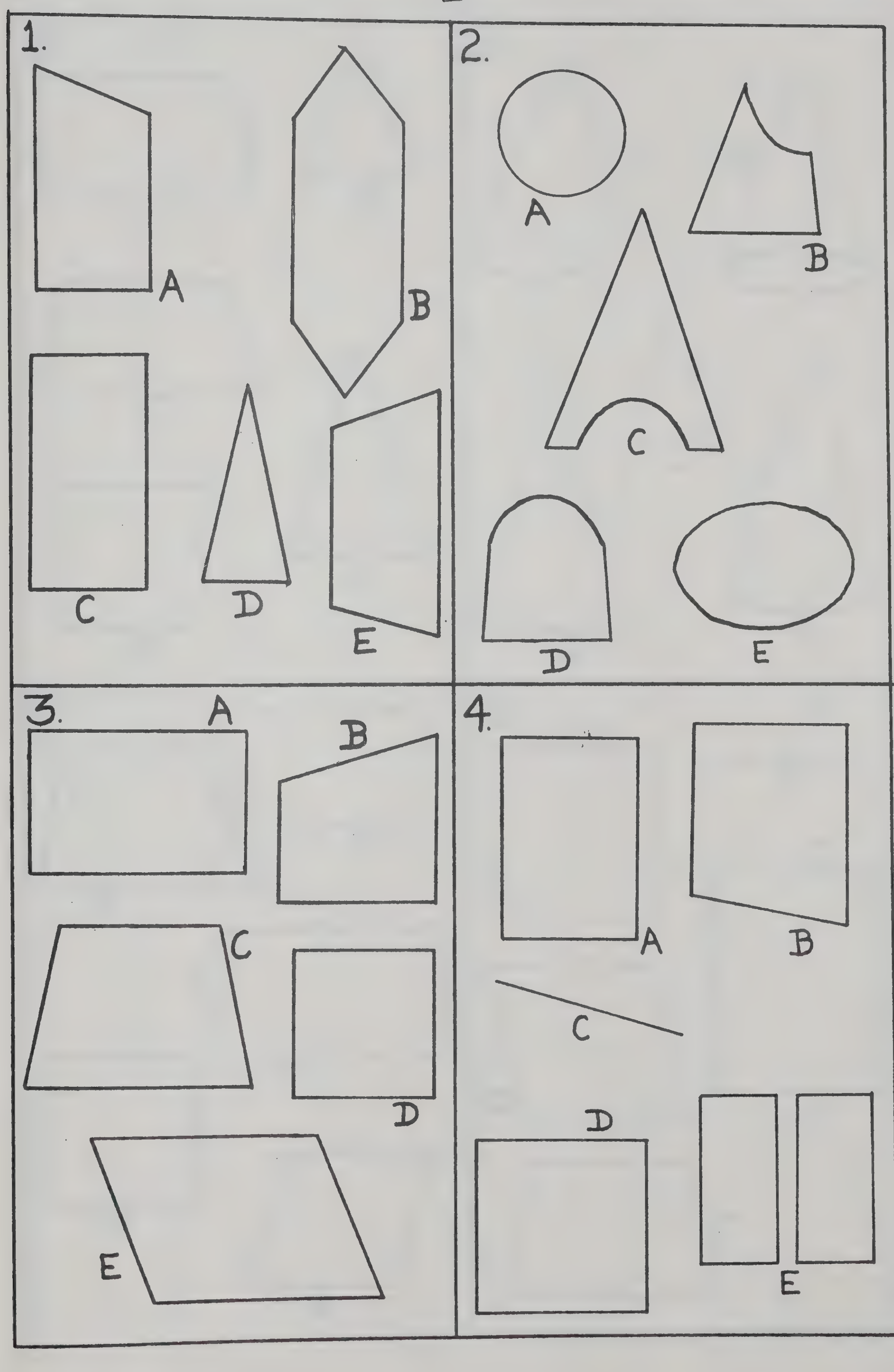


16. cube: oblique

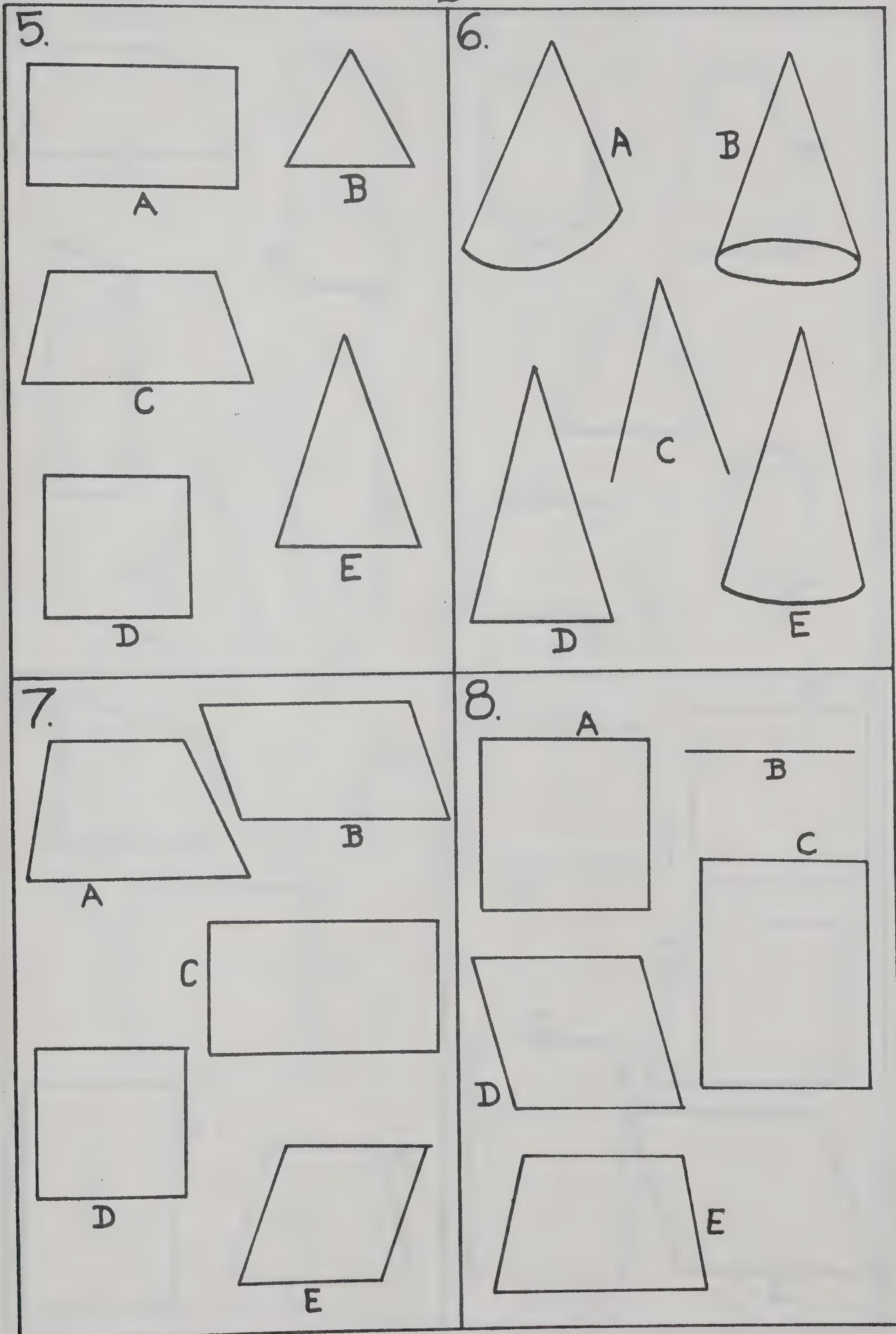


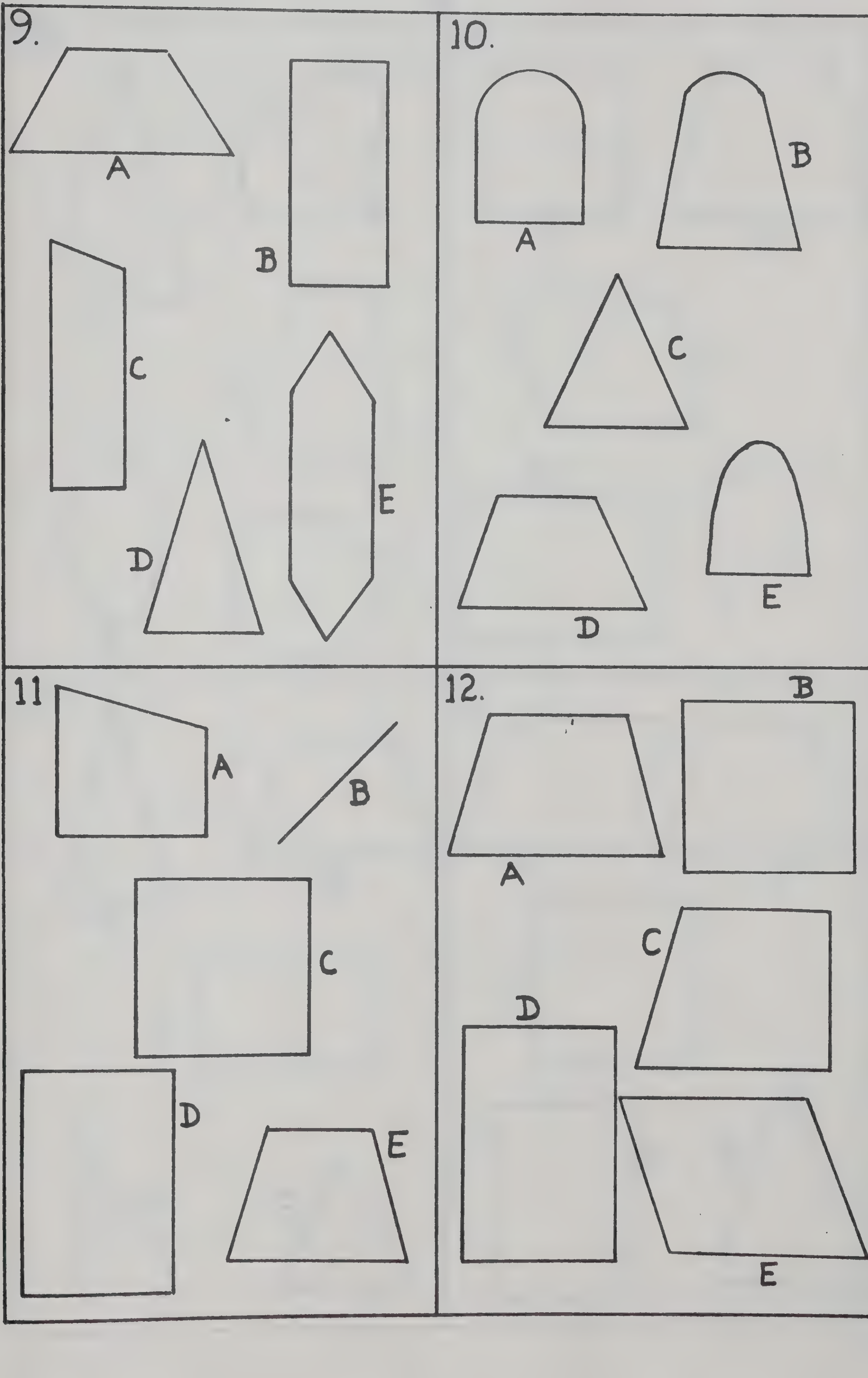


1

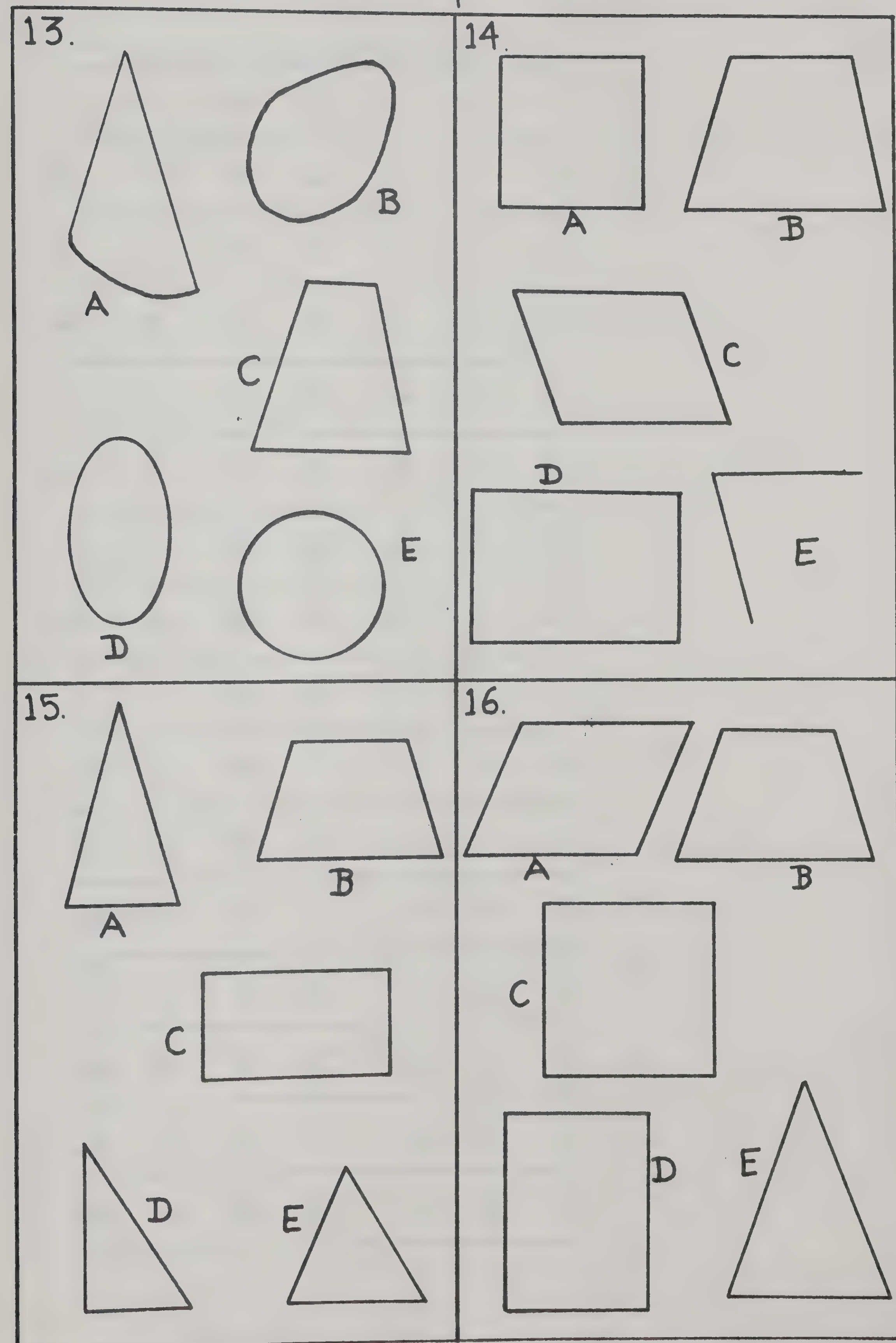


2




9.
A
B
C
D
E
10.
A
B
C
D
E
11
A
B
C
D
E
12.
A
B
C
D
E

4



1.	A	B	C	D	E	F
2.	A	B	C	D	E	F
3.	A	B	C	D	E	F
4.	A	B	C	D	E	F
5.	A	B	C	D	E	F
6.	A	B	C	D	E	F
7.	A	B	C	D	E	F
8.	A	B	C	D	E	F
9.	A	B	C	D	E	F
10.	A	B	C	D	E	F
11.	A	B	C	D	E	F
12.	A	B	C	D	E	F
13.	A	B	C	D	E	F
14.	A	B	C	D	E	F
15.	A	B	C	D	E	F
16.	A	B	C	D	E	F

CORRECT ANSWERS

DRAWING		MULTIPLE CHOICE		
1.	parallelogram	1.	C	rectangle
2.	rectangle	2.	A	circle
3.	square	3.	E	parallelogram
4.	ellipse	4.	D	square
5.	equilateral triangle	5.	F	none of these
6.	parallelogram	6.	D	isosceles triangle
7.	isosceles triangle	7.	B	parallelogram
8.	square	8.	A	square
9.	isosceles triangle	9.	B	rectangle
10.	parallelogram	10.	E	one-half hyperbola
11.	circle	11.	C	square
12.	square	12.	E	parallelogram
13.	parallelogram	13.	D	ellipse
14.	rectangle	14.	C	parallelogram
15.	rectangle	15.	E	equilateral triangle
16.	one-half hyperbola	16.	D	rectangle

APPENDIX D

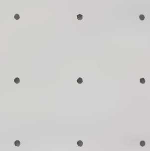
PRE-TEST

CREATIVE GEOMETRY TEST I

(CG1)

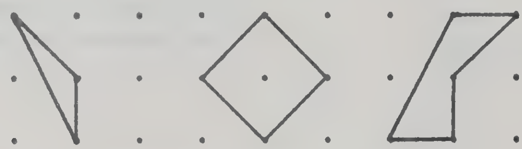
CREATIVE GEOMETRY TEST I (CG1)

1) You are given a region with 9 dots:

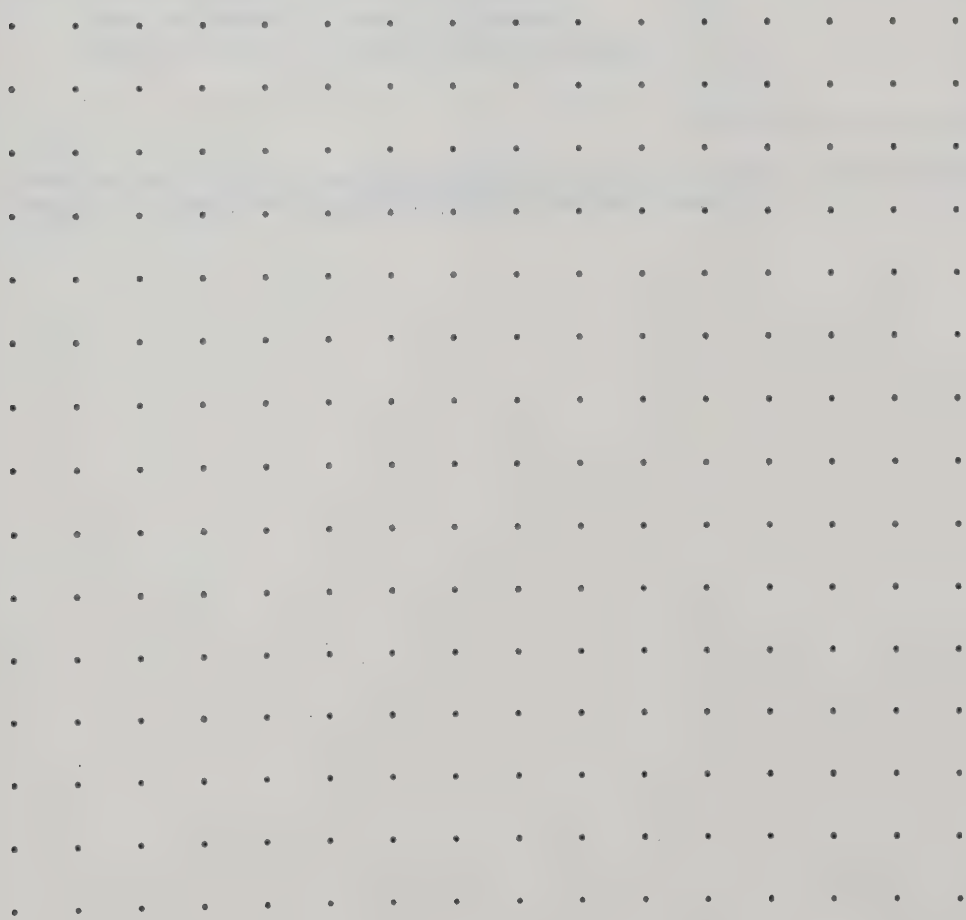


You are asked to draw as MANY DIFFERENT closed geometrical figures as you can think of. The end-points (vertices) of your figures must lie on the dots.

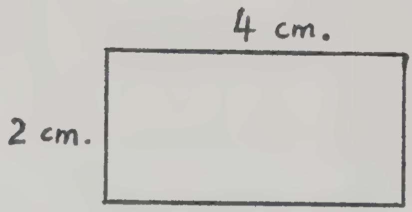
EXAMPLES:



DO NOT DRAW ALL YOUR FIGURES ON ONE REGION!
USE A NEW 9-DOT REGION FOR EACH DIFFERENT FIGURE!!



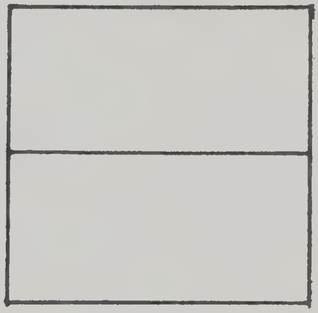
2) This is a RECTANGLE:



You are asked to say something about this rectangle by doing something about it.

EXAMPLES:

a) A square can be formed by putting 2 such rectangles together:



b) The rectangle can be separated into a 3-sided figure (triangle) and a 4-sided figure (quadrilateral):

Now go on to write as MANY IDEAS as you can about this rectangle. DRAW A FIGURE FOR EACH STATEMENT YOU PUT DOWN!!

APPENDIX E

POST-TEST

TRADITIONAL MOTION GEOMETRY TEST

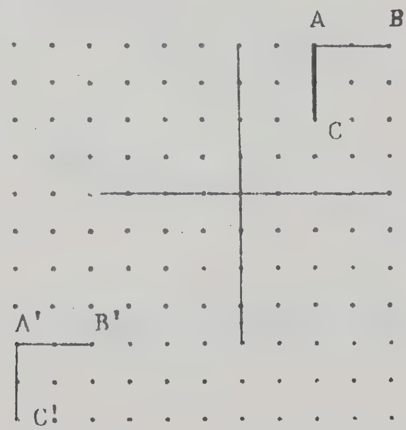
(TMG)

1. One property of a slide is that

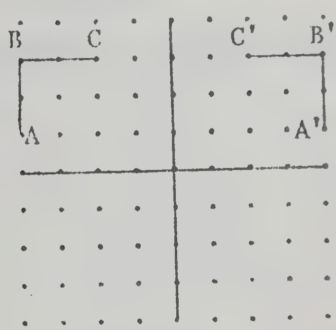
   A. an original and image are congruent with respect to the whole figure as well as the parts.
   B. line segment and image are parallel.
   C. line segments joining points and their images are congruent and parallel.
   D. all of the above.

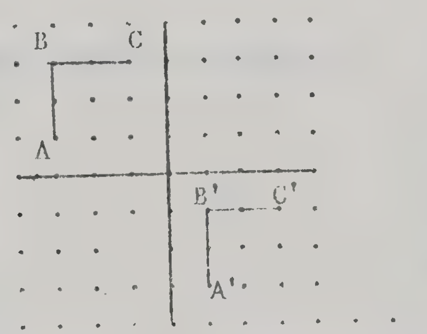
2. The diagram illustrates a

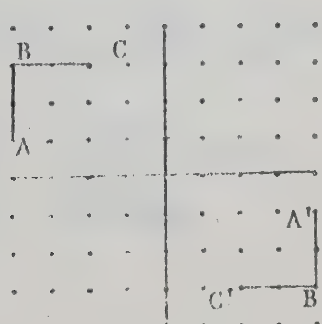
   A. Rotation
   B. Flip
   C. Slide
   D. Reflection

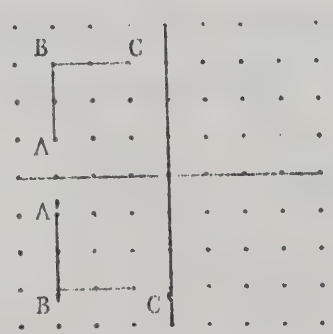


3. The diagram that illustrates a slide is

A.


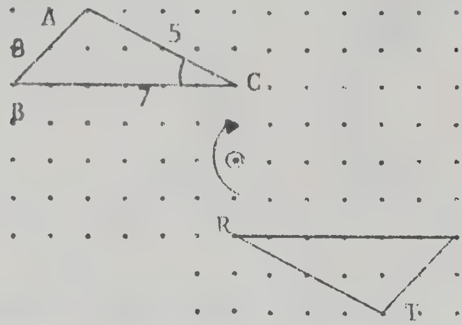
B.


C.


D.


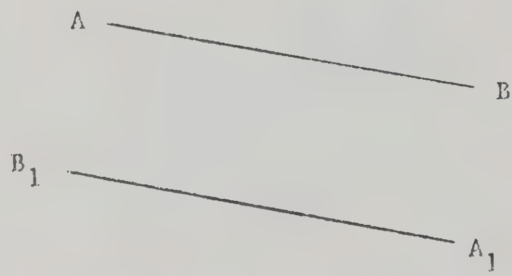
4. The following statement is true about the image and the original

A. $\overline{RT} = 3$ units
B. $\angle A \cong \angle S$
C. $\angle C \cong \angle R$
D. $\overline{RS} \cong \overline{AC}$



5. The image is obtained by

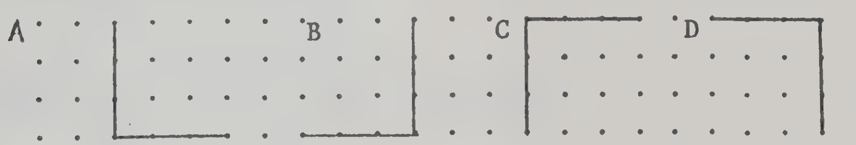
A. a slide
B. a rotation
C. a reflection
D. a flip

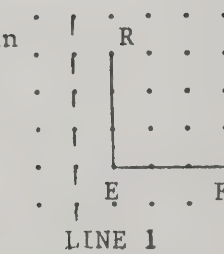


6. A segment joining a point and its half turn image is bisected by

A. a slide
B. a reflection
C. a turn center
D. a glide

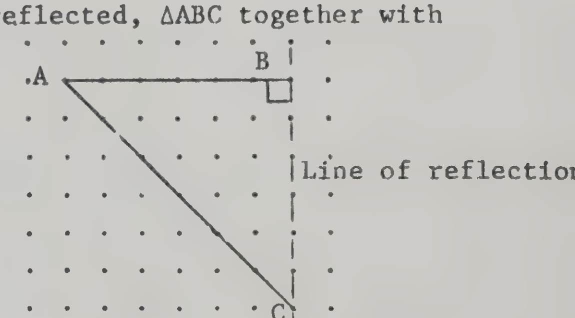
7. If REF is reflected about line 1 the image would be as shown in





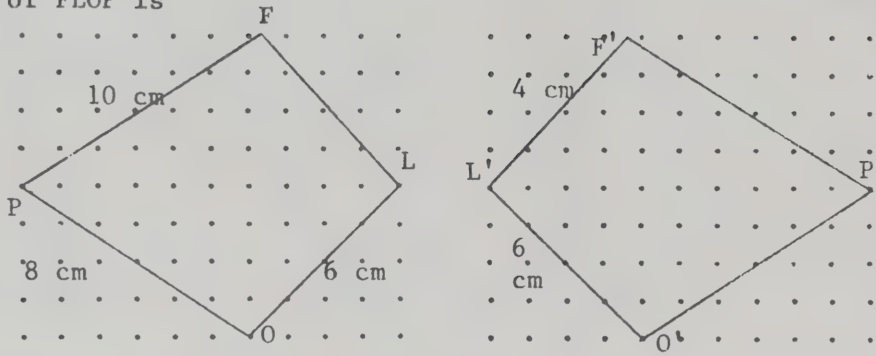
8. ΔABC is a right triangle. If ABC is reflected, ΔABC together with its image will form

A. a rhombus
B. a parallelogram
C. a square
D. a triangle



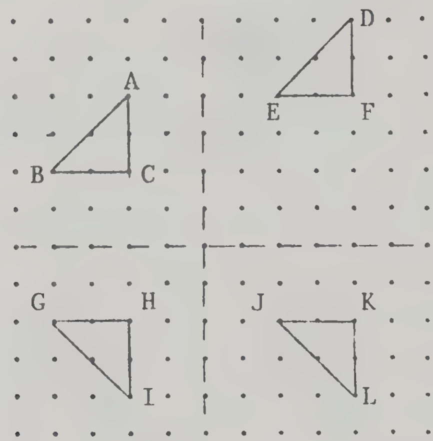
9. The perimeter of FLOP is

A. 24 cm
B. 32 cm
C. 28 cm
D. 34 cm



10. The figure which is a slide reflection of ABC is

A. figure DEF
B. figure JKL
C. figure GHI
D. none of the above

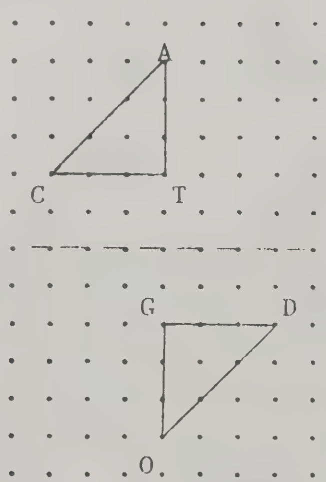


11. A slide reflection is a combination of a

A. slide and a ¼ turn
B. slide and a glide
C. a flip and a turn
D. slide and a reflection

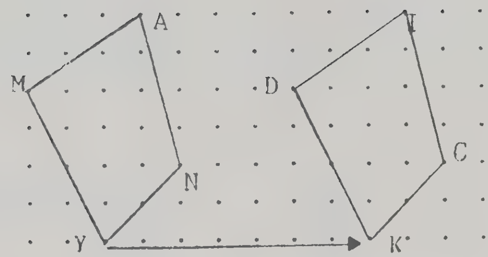
12. ΔDOG is the image of ΔCAT formed by a

A. slide reflection
B. reflection and ½ turn
C. slide and ½ turn
D. two slides



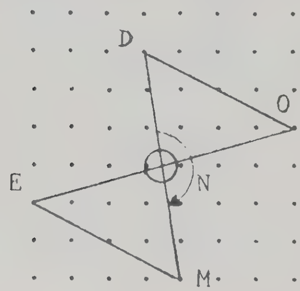
13. ∠MAN is congruent to

A. ∠KCI
B. ∠CKD
C. ∠DIC
D. ∠DKC



14. ∠DON is congruent to

A. ∠ENM
B. ∠MEN
C. ∠EMN
D. ∠NME

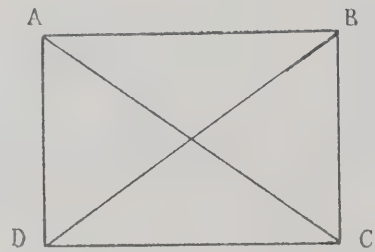


15. A hexagon has ________ diagonals

A. 6
B. 7
C. 8
D. 9

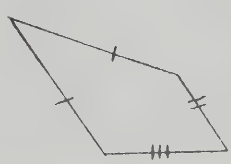
16. A diagonal of polygon ABCD is

A. $\overline{AB}$
B. $\overline{BC}$
C. $\overline{AD}$
D. $\overline{AC}$

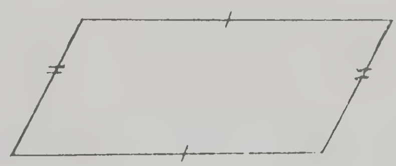


17. The diagram which is a regular polygon is

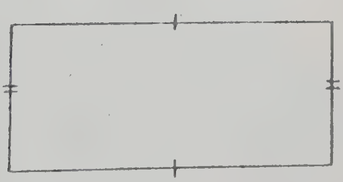
A.



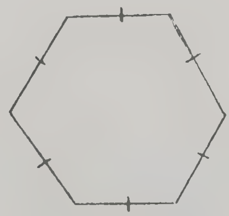
B.



C.



D.



18. The segment AB and its ¼ turn produces

A. a right angle
B. a straight angle
C. an acute angle
D. an obtuse angle

19. The angle which is an obtuse angle is

A. 40°

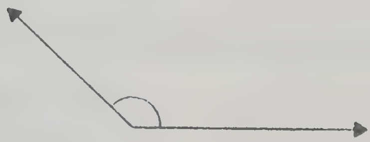
B. 90°

C. 150°

D. 200°

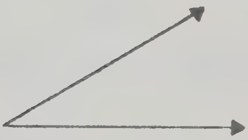
20. The figures which is an obtuse angle is

A.



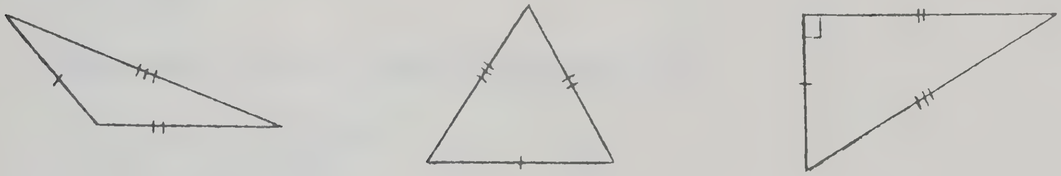
B.

C.



D.

21. Name the triangles below according to their angles from left to right



A. acute, right, equilateral
B. obtuse, acute, right
C. acute, scalene, right
D. obtuse, right, scalene

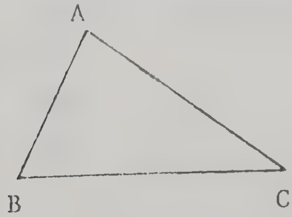
22. An equilateral traingle is also

A. an obtuse triangle
B. a right triangle
C. an isosceles triangle
D. a scalene triangle

23. An obtuse triangle has

A. three obtuse angles
B. two obtuse angles and one acute angle
C. two obtuse angles and one right angle
D. one obtuse angle and two acute angles

24. By inspection and according to sides, the BEST name for ΔABC would be



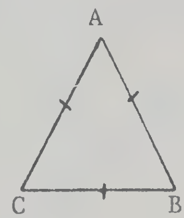
A. obtuse triangle
B. right triangle
C. scalene triangle
D. isosceles triangle

25. A rectangular piece of paper is folded so that the crease forms a diagonal of the rectangle. The triangles developed are, according to sides

A. right triangles
B. isosceles triangles
C. scalene triangles
D. obtuse triangles

26. The BEST name for $\triangle ABC$ classified by its sides would be

A. equilateral
B. equiangular
C. scalene
D. isosceles



27. A triangle with no lines of symmetry is

A. equilateral
B. isosceles
C. scalene
D. symmetrical

28. A triangle which has no lines of symmetry has

A. three angles congruent
B. two angles congruent
C. two sides congruent
D. no sides congruent

29. Each line of symmetry separates the figure into two congruent right triangles for all

A. isosceles triangles
B. right triangles
C. scalene triangles
D. obtuse triangles

30. If two angles of a triangle measure 25° and 35° each then the third angle

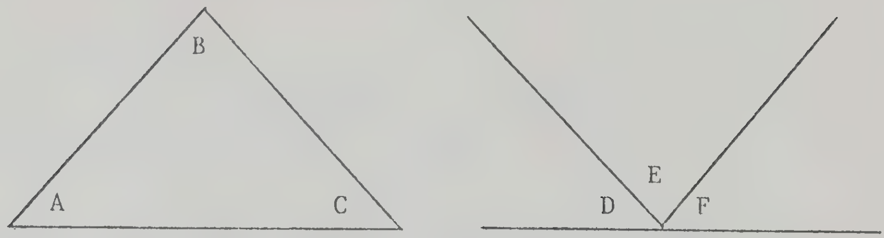
A. cannot be determined
B. will be acute
C. cannot be over 90°
D. will be 120°

31. If two angles of a triangle measure 55° and 15°, then the third angle measures

A. cannot be determined
B. 20°
C. 110°
D. 290°

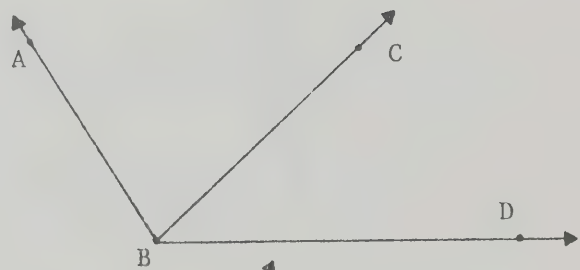
32. If $\angle A \cong \angle F = 50^\circ$, $\angle C \cong \angle D = 45^\circ$, $\angle B \cong \angle E =$ ______

A. 75°
B. 80°
C. 85°
D. 90°



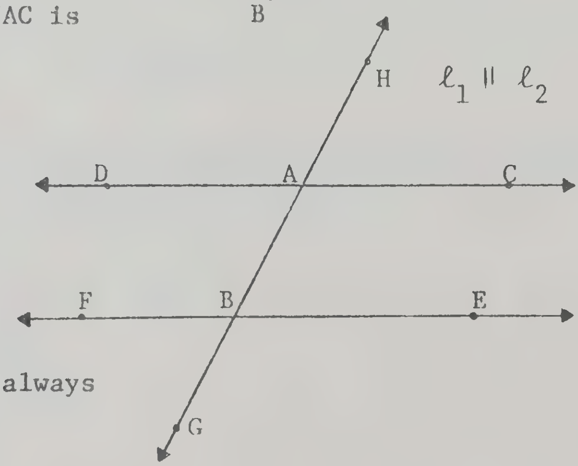
33. $\angle$ ABC and $\angle$ CBD are

A. opposite angles
B. complementary angles
C. supplementary angles
D. adjacent angles



34. The supplement of $\angle$ HAC is

A. $\angle$ DAB
B. $\angle$ FBG
C. $\angle$ ABE
D. $\angle$ HAD

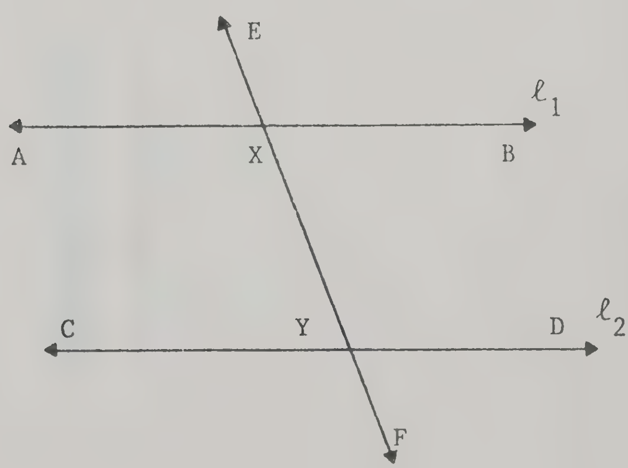


35. Opposite angles are always

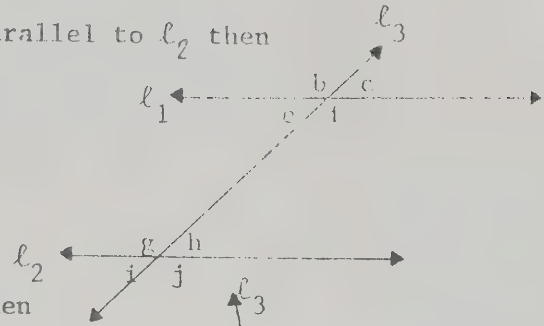
A. congruent angles
B. acute angles
C. obtuse angles
D. supplementary angles

36. $\angle$ EXA and $\angle$ XYC are called

A. supplementary angles
B. alternate angles
C. opposite angles
D. corresponding angles



37. If $\angle b = 145^{\circ}$, and ℓ_1 is parallel to ℓ_2 then



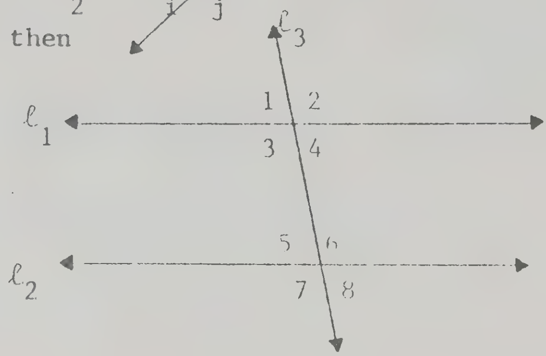
A. $\angle i = 145^{\circ}$

B. $\angle j = 145^{\circ}$

C. $\angle h = 145^{\circ}$

D. $\angle e = 145^{\circ}$

38. If $\angle 4 = 75^{\circ}$ and $\ell_1 \parallel \ell_2$ then



A. $\angle 2 = 75^{\circ}$

B. $\angle 3 = 75^{\circ}$

C. $\angle 5 = 75^{\circ}$

D. $\angle 7 = 75^{\circ}$

39. The distance around the sides of a polygon is called

A. area
B. surface area
C. volume
D. perimeter

40. To calculate the perimeter of <u>any</u> polygon use the formula

A. P = lw
B. P = a + b + c
C. P = ns
D. P = sum of sides

<u>KEY</u>:

1.	D	11.	3	21.	B	31.	C
2.	C	12.	B	22.	C	32.	C
3.	B	13.	C	23.	D	33.	D
4.	C	14.	B	24.	C	34.	D
5.	B	15.	D	25.	C	35.	A
6.	C	16.	D	26.	A	36.	D
7.	B	17.	D	27.	C	37.	B
8.	D	18.	A	28.	D	38.	C
9.	C	19.	C	29.	A	39.	D
10.	B	20.	A	30.	D	40.	D

APPENDIX F

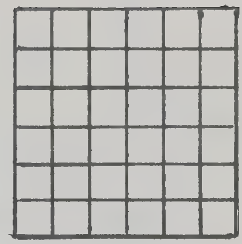
POST-TEST

CREATIVE GEOMETRY TEST II

(CG2)

CREATIVE GEOMETRY TEST II (CG2)

1) You are given a SQUARE:

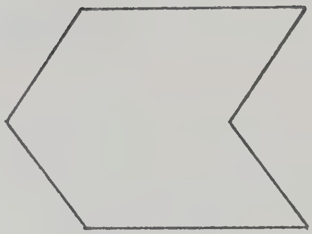


Cut the square into 2 parts of EQUAL AREAS.

You are asked to DRAW ALL THE POSSIBLE WAYS OF CUTTING by copying the square onto the given graph paper.

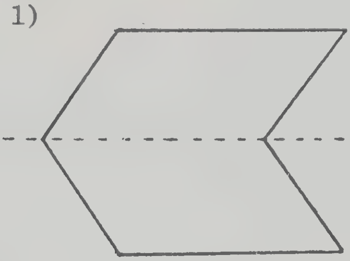
Draw as MANY CUTS as you can think of.

2) This is a HEXAGON (a 6-sided polygon):



You are asked to SAY SOMETHING about this hexagon by DOING SOMETHING about it.

EXAMPLE: 1)



The hexagon can be divided into 2 different quadrilaterals (4-sided polygons); in fact they are parallelograms.

Now go on to write as MANY IDEAS as you can think of about this hexagon.

DRAW A FIGURE FOR EACH STATEMENT YOU PUT DOWN.

APPENDIX G

POST-TEST

CREATIVE MOTION GEOMETRY TEST

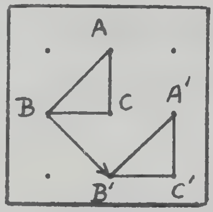
(CMG)

CREATIVE MOTION GEOMETRY TEST (CMG)

1) You are given a closed region with 9 dots and $\triangle$ ABC.

You can form MANY IMAGES of $\triangle$ ABC through a slide, or a turn, or a reflection, or a slide reflection, or a combination of glides.

EXAMPLE:



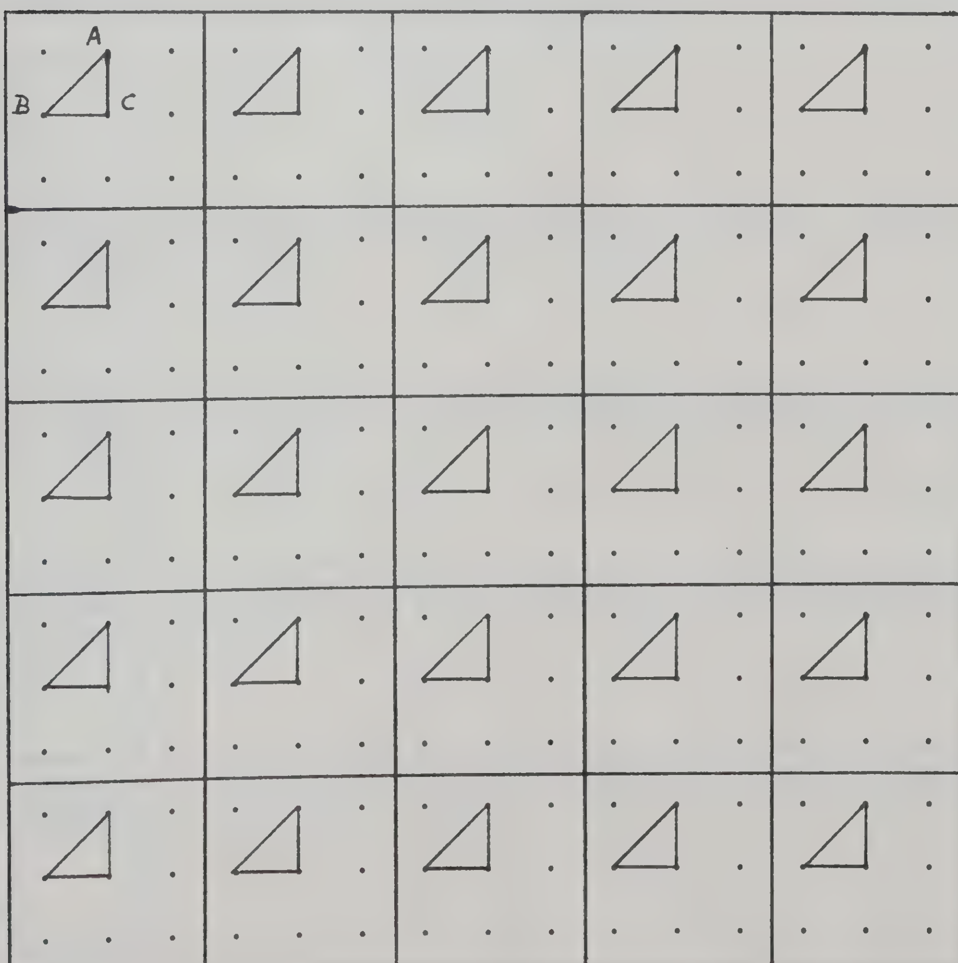
You are asked to form ALL THE POSSIBLE DIFFERENT IMAGES of $\triangle$ ABC WITHIN THE 9-DOT REGION.

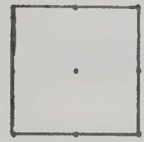
DO NOT DRAW ALL YOUR IMAGES ON ONE REGION.

Use a new 9-dot region for each different image.

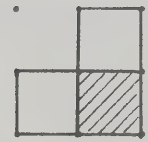
Indicate your motion with appropriate notations:

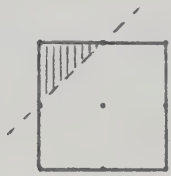
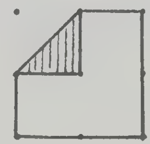
-------- ; ——————→ ; or  .



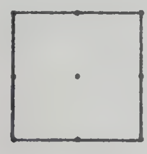
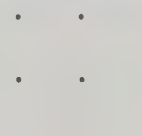
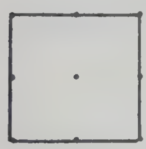
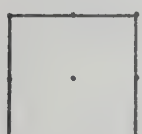
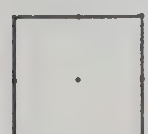
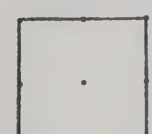
2) You are given a SQUARE on a 9-DOT REGION: 

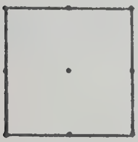
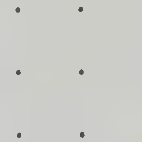
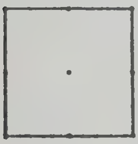
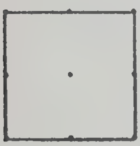
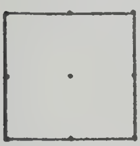
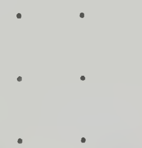
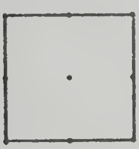
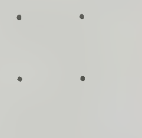
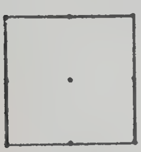
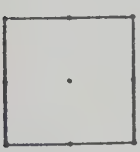
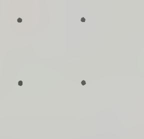
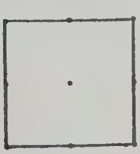
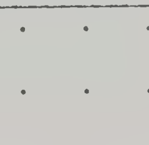
By selecting your own motion, you can change the square into some other polygon.

EXAMPLES: 1)  Slide shaded part (1R,1D); new polygon: 

2)  Reflect shaded part in the mirror line; new polygon: 

Now, select your own SHADED PART, and MOTION (slide, turn, reflection, slide-reflection, or combination of glides). Write down all your different results in the following chart:

SHADED PART	MOTION	RESULTED POLYGON
		
		
		
		
		

SHADED PART	MOTION	RESULTED POLYGON
		
		
		
		
		
		
		
		

APPENDIX H

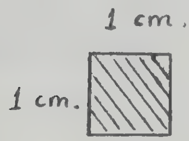
POST-TEST

TRANSFER AREA TEST

(TAR)

TRANSFER AREA TEST (TAR)

In order to measure AREA, we use a UNIT of area which is a SQUARE of side 1 unit. For this set of exercises, our unit is 1 $cm.^2$



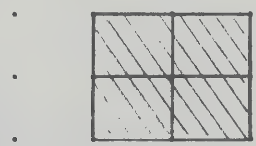
AREA = $1 \times 1\ cm.^2$

$= 1\ cm.^2$

Find the area of each of the shaded regions.

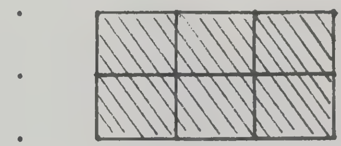
Show all the steps of your calculation.

Two worked examples are given as hints:



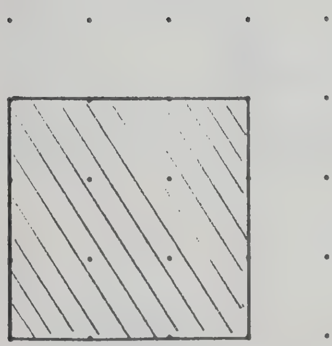
a) Area = $2 \times 2\ cm.^2$

$= 4\ cm.^2$

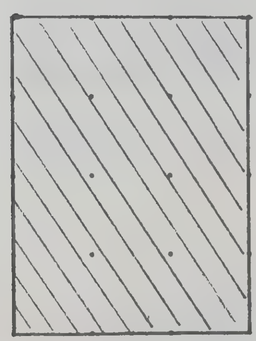


b) Area = $2 \times 3\ cm.^2$

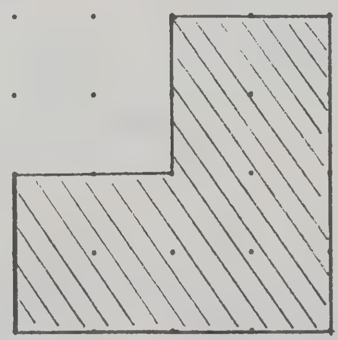
$= 6\ cm.^2$



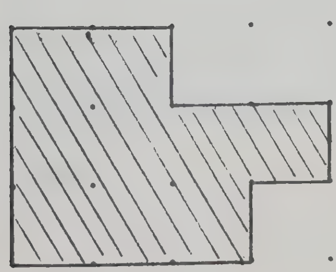
AREA =



AREA =

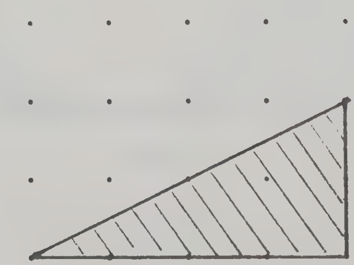


AREA =



AREA =

AREA =



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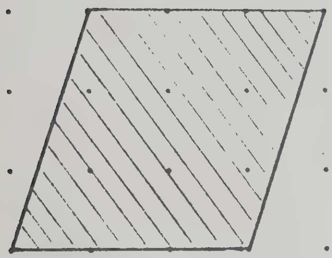

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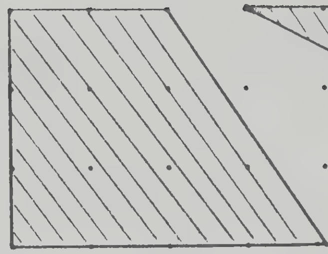

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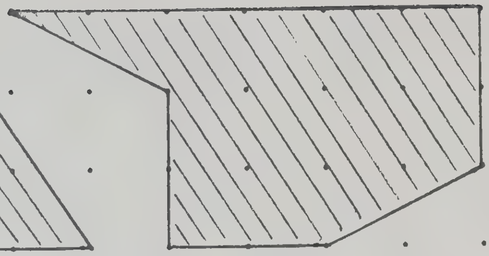

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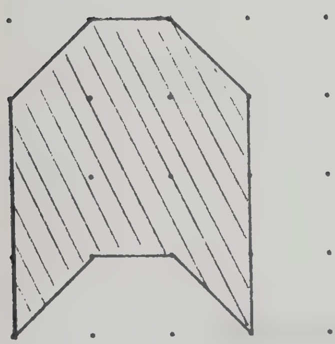

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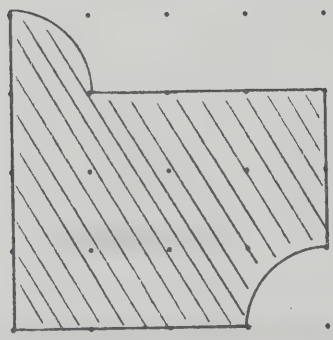

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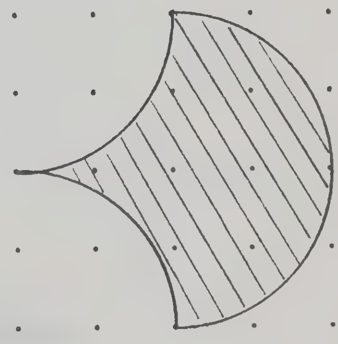
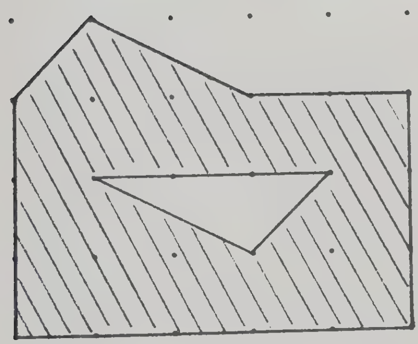
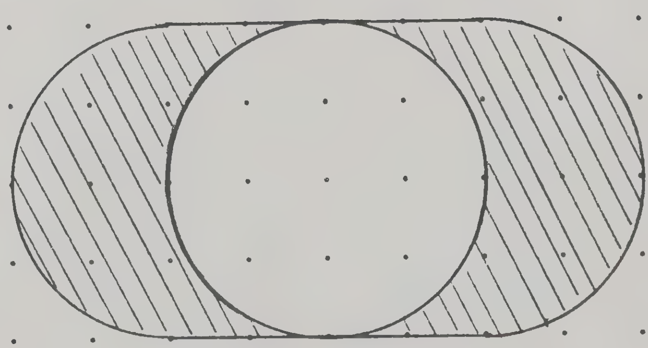


AREA =



AREA =



AREA =

APPENDIX I

STATISTICAL ANALYSES OF GROUP DIFFERENCES

INCLUDING STUDENTS WITH INCOMPLETE DATA

STATISTICAL ANALYSIS OF GROUP DIFFERENCES INCLUDING STUDENTS WITH INCOMPLETE DATA

The initial sample of the present study consisted of 58 students (30 in the experimental group and 28 in the control group). After elimination of those students with incomplete data, the final sample analysed included a total of 41 students (experimental, n=20; control, n=21). It is possible, of course, that the 17 students omitted from analysis might have contributed to the quality of classroom discussions (ID and TD) which were significant components of the instructional methods (IM and TM). To account for this possibility, we should therefore determine whether the two groups (IM and TM) of students, including these 17 students, were compatible with respect to all the 5 pretests (GR7, A5T, MAR, SEC and CG1).

Separate univariate t-tests were performed on GR7, A5T, MAR and SEC, owing to the uneven number of students taking these tests in the control group. The results presented in Table 9.1 show that none of the t-values are significant at the .05 level. Hotelling's T^2 test was conducted on the fluency, diversity and rarity scores of CG1. Table 9.2 indicates that the assumption of homoscedasticity is tenable, with the computed T^2 being insignificant at the .05 level.

These results therefore demonstrate the compatibility of the two groups, including those students without complete data, with respect to students' mathematical background, geometric

maturity and mathematical creativity. We can reasonably assume that, considering the students of each group as a whole, the students' effects on the teaching-learning situation of their respective groups were equivalent.

TABLE 9.1

t-TESTS ON MEAN SCORES OF 4 PRETESTS (GR7, A5T, MAR, SEC) FOR INVENTIVE AND TRADITIONAL GROUPS, INCLUDING STUDENTS WITH INCOMPLETE SCORES ON PRETESTS AND POSTTESTS.

	INVENTIVE		TRADITIONAL			
TESTS	X	N	X	N	t	P
GR7	65.0	30	66.4	26	.29	.77
A5T	60.0	30	64.7	27	.82	.42
MAR	64.8	30	67.0	25	.39	.70
SEC	19.9	29	18.9	28	.70	.49

TABLE 9.2

HOTELLING'S T^2-TEST ON MEANS OF PRETEST SCORES ON CREATIVE GEOMETRY TEST I (CG1), INCLUDING STUDENTS WITH INCOMPLETE DATA.
(N=57)

GROUP	N	CG1 Fluency	CG1 Diversity	CG1 Rarity	T^2	F(3, 53)	P
INVENTIVE	30	27.6	18.2	9.7	1.95	.63	.60
TRADITIONAL	27	27.2	19.0	10.7			

Bartlett-Box Tests of Homogeneity of Variances:

DF = 6, χ^2 = 10.63, P = .101

F(6, 21236.3) = 1.77 P = .101

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